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OPEN AND DISTANCE LEARNING (ODL) PROGRAMMES

(FOR THOSE WHO JOINED THE PROGRAMMES FROM THE ACADEMIC YEAR 2023–2024)

M. Sc. Physics

Course Material

Nuclear and Particle Physics

SPHM41

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SYLLABUS- NUCLEAR AND PARTICLE PHYSICS

UNIT I: NUCLEAR MODELS

Liquid drop model – Weizacker mass formula – Isobaric mass parabola–Mirror Pair - Bohr Wheeler theory of fission – shell model – spin-orbit coupling – magic numbers – angular momenta and parity of ground states – magnetic moment – Schmidt model – electric Quadrupole moment - Bohr and Mottelson collective model – rotational and vibrational bands.

UNIT II: NUCLEAR FORCES

Nucleon – nucleon interaction – Tensor forces – properties of nuclear forces – ground state of deuteron – Exchange Forces - Meson theory of nuclear forces – Yukawa potential – nucleon-nucleon scattering –effective range theory – spin dependence of nuclear forces – charge independence and charge symmetry – isospin formalism.

UNIT III: NUCLEAR REACTIONS

Kinds of nuclear reactions – Reaction kinematics – Q-value – Partial wave analysis of scattering and reaction cross section – scattering length– Compound nuclear reactions – Reciprocity theorem – Resonances –Breit Wigner one level formula – Direct reactions - Nuclear Chain reaction – four factor formula.

UNIT IV: NUCLEAR DECAY

Beta decay – Continuous Beta spectrum – Fermi theory of beta decay - Comparative Half-life –Fermi Kurie Plot – mass of neutrino – allowed and forbidden decay — neutrino physics – Helicity - Parity violation - Gamma decay – multipole radiations – Angular Correlation – internal conversion – nuclear isomerism – angular momentum and parity selection rules.

UNIT V: ELEMENTARY PARTICLES

Classification of Elementary Particles – Types of Interaction and conservation laws – Families of elementary particles – Isospin – Quantum Numbers – Strangeness – Hypercharge and Quarks –SU (2) and SU (3) groups-Gell Mann matrices– Gell Mann Okuba Mass formula-Quark Model. Standard model of particle physics – Higgs boson

UNIT I: NUCLEAR MODELS

Liquid drop model – Weizacker mass formula – Isobaric mass parabola–Mirror Pair - Bohr Wheeler theory of fission – shell model – spin-orbit coupling – magic numbers – angular momenta and parity of ground states – magnetic moment – Schmidt model – electric Quadrupole moment - Bohr and Mottelson collective model – rotational and vibrational bands.

Liquid Drop Model

Bethe- Weizsacker in 1935 proposed on the basis of experimental facts that a nucleus resembles a drop of liquid. In 1939, Bohr and Wheeler further developed this model to explain the phenomenon of nuclear fission.
Following are some of the similarities between a drop of liquid and nucleus, which prompted Weizsacker to develop the liquid drop model.

Similarities between Liquid Drop and Nucleus

1. Nuclear forces are analogous to the surface tension of a liquid.

2. The nucleons behave in a manner similar to that of molecules in a liquid drop.

3. The density of the nuclear matter is almost independent of A, showing resemblance to liquid drop where the density of a liquid is independent of the size of the drop.

4. The constant binding energy per nucleon is analogous to the latent heat of vaporization.

5. The disintegration of nuclei by the emission of particles is analogous to the evaporation of molecules from the surface of liquid.

6. The absorption of bombarding particles by a nucleus corresponds to the condensation of drops.

7. The energy of nuclei corresponds to internal thermal vibrations of drop molecules.

These similarities led to the Liquid Drop Model, which emphasized strong internucleon attraction while ignoring finer nuclear force details.

Assumptions of the Liquid Drop Model

1. The nucleus consists of incompressible matter.
2. The nuclear force is identical for every nucleon.
3. The nuclear force saturates.
4. In an equilibrium state, the nuclei of atom remain spherically symmetric under the action of strong attractive nuclear forces.

Weizsacker Mass Formula

The Weizsacker's semiempirical mass formula (SEMF) is based on the liquid drop model of the nucleus, where the nucleus is considered similar to a charged liquid drop. This analogy helps estimate the mass and binding energy of a nucleus in its ground state. The formula accounts for various factors influencing nuclear stability and is given by the sum of five key energy terms:

1. Volume Energy (B_1)

This term arises from the strong nuclear force, which binds nucleons together. Each nucleon interacts only with its nearest neighbors, making the binding energy proportional to the total number of nucleons (**A**).

$$B_1 - a_v A$$

where a_v is a proportionality constant.

2. Surface Energy (B_2)

Nucleons at the surface experience fewer interactions than those in the interior, leading to a reduction in binding energy. This correction term is proportional to the surface area of the nucleus, which scales as $\mathbf{A^{2/3}}$.

$$B_2 = -a_s A^{2/3}$$

The negative sign indicates a decrease in binding energy.

3. Coulomb Energy (B_3)

Protons repel each other due to Coulomb repulsion, reducing nuclear stability. This effect depends on the number of protons **Z** and the nuclear radius (**$r \approx r_0A^{1/3}$**), leading to:

$$B_3 = -a_c\frac{Z^2}{A^{1/3}}$$

where a_c is a constant.

4. Asymmetry Energy (B_4)

Nuclei are most stable when **Z ≈ N** (i.e., equal numbers of protons and neutrons). A large difference between **Z** and **N** decreases binding energy, making the nucleus less stable. This term is proportional to **$(N - Z)^2/A$**:

$$B_4 = -a_a\frac{(N-Z)^2}{A}$$

where a_a is a constant.

5. Pairing Energy (B_5)

Nuclei with even numbers of protons and neutrons are more stable due to nucleon pairing effects. This term depends on whether **Z** and **N** are even or odd:

$$B_5 = \begin{cases} +a_p, & \text{even-even nuclei} \\ 0, & \text{odd A nuclei} \\ -a_p, & \text{odd-odd nuclei} \end{cases}$$

where a_p is a pairing constant.

Number of stable isotopes

Z	N	Number of stable nuclei
Even	Even	165
Even	Odd	55
Odd	Even	50
Odd	Odd	5

(the five stable odd Z-odd N nuclei are: ${}^{2}_{7}$H, ${}^{6}_{3}$Li, ${}^{10}_{5}$ B, ${}^{14}_{7}$ N, ${}^{180}_{73}$Ta)

Final Semi empirical Mass Formula:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm a_p$$

This formula provides an approximation for nuclear binding energy, balancing the effects of nuclear forces, Coulomb repulsion, and quantum stability factors.

$$\begin{aligned} a_v &= 15.5\text{MeV} \\ a_s &= 16.8\text{MeV} \\ a_c &= 0.7\text{MeV} \\ a_a &= 23.0\text{MeV} \\ a_p &\begin{cases} -34\text{MeV} & \text{for even-even nuclei} \\ = 0\text{MeV} & \text{for odd } A \text{ nuclei} \\ --34\text{MeV} & \text{for odd-odd nuclei} \end{cases} \end{aligned}$$

Achievements of Liquid Drop Model

• It predicts the atomic masses and binding energies of various nuclei accurately.

• It predicts emission of a- and b-particles in radioactivity.

• The theory of compound nucleus, which is based on this model, explains the basic features of the fission process.

Failures of Liquid Drop Model

• It fails to explain the extra stability of certain nuclei, where the numbers of protons or neutrons in the nucleus are 2, 8, 20, 28, 50, 82 or 126 (these numbers are called magic numbers).

• It fails to explain the measured magnetic moments of many nuclei.

• It also fails to explain the spin of nuclei.

• It is also not successful in explaining the excited states in most of the nuclei.

• The agreement of semiempirical mass formula with experimentally observed masses and binding energies is poor for lighter nuclei compared to the heavy ones.

Isobaric Mass Parabola

The isobaric mass parabola explains nuclear stability trends and beta decay pathways, helping predict the most stable isotopes for a given mass number A.

The isobaric mass parabola represents the variation of nuclear mass (or binding energy) as a function of Z (proton number) for a fixed A (mass number). It is derived from the semiempirical mass formula, particularly focusing on the Coulomb and asymmetry energy terms, which influence nuclear stability.

Expression for Mass of an Isobar

The mass of a nucleus M(Z, A) is given by:

$$M(Z, A) = Zm_p + (A - Z)m_n - B(Z, A)/c^2$$

where B(Z, A) is the binding energy. Substituting the semiempirical mass formula, the dependence on Z becomes:

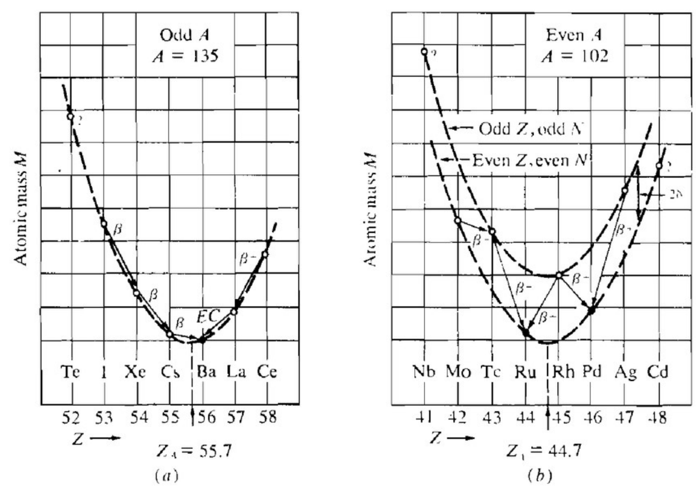
$$M(Z,A) = C - a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A}$$

where C is a constant independent of Z.

Rearranging the equation:

$$M(Z,A) = C' + aZ^2 - bZ$$

where a and b are constants. This equation represents a parabola in terms of Z, showing that for a given A, nuclear mass follows a quadratic curve.



Key Features

1. Most Stable Isobar: The nucleus with the lowest mass (or highest binding energy) lies at the minimum of the parabola, determined by:

$$Z_{\text{stable}} \approx \frac{A}{2}\left(1 + \frac{a_c A^{2/3}}{4a_a}\right)$$

2. Beta Decay: If an isobar has a mass higher than its neighbouring isobar, it undergoes β-decay to move toward the minimum of the parabola, achieving greater stability.
3. Odd-A vs. Even-A Isobars:

For even A, a single parabola appears, with stability at the minimum. For odd A, two closely spaced parabolas exist due to pairing energy, causing isobars with even Z to be slightly more stable.

Mirror Pair

In nuclear physics, a mirror pair (or mirror nuclei) consists of two nuclei where the number of protons and neutrons are interchanged. That is, if one nucleus has Z protons and N neutrons, its mirror partner will have N protons and Z neutrons. Properties of Mirror Nuclei

1. Same Mass Number (A): Since they have the same total number of nucleons (A=Z+N), their atomic masses are nearly identical.
2. Charge Symmetry: Mirror nuclei exhibit charge symmetry in nuclear forces because strong nuclear interactions are charge-independent.
3. Energy Levels: The energy levels of mirror nuclei are often similar but slightly shifted due to Coulomb interactions (since the proton count differs).
4. Beta Decay Relationship: If one of the mirror nuclei is unstable, it can undergo beta decay to transform into its mirror partner.

Mirror Nuclei

$^{3}_{1}H$ (Tritium) and $^{3}_{2}He$ (Helium-3)

$^{7}_{3}Li$ (Lithium-7) and $^{7}_{4}Be$ (Beryllium-7)

$^{2}1_{10}Ne$ (Neon-21) and $^{2}1_{11}Na$ (Sodium-21)

Significance of Mirror Nuclei

- They provide insights into isospin symmetry in nuclear physics.
- They help in testing the charge symmetry and charge independence of nuclear forces.

Bohr-Wheeler theory of Fission

In 1939, Bohr and J.A. Wheeler proposed a theory of nuclear fission using the liquid drop model. This model describes the nucleus as a charged liquid drop, where mechanical vibrations can cause deformation, potentially leading to fission.

When a nucleus absorbs energy, such as through neutron absorption, it gains excitation energy, generating vibrations. These vibrations modify the surface energy (Es) and electrostatic energy (Ec), influencing nuclear stability.

During fission, the nucleus undergoes severe deformation. Surface tension tries to restore the spherical shape, while electrostatic repulsion increases deformation. If the repulsive force overcomes surface tension, the nucleus splits into two fragments, releasing energy. The deformation process leading to fission resembles the breakup of a liquid drop and can be calculated using the activation energy for fission.

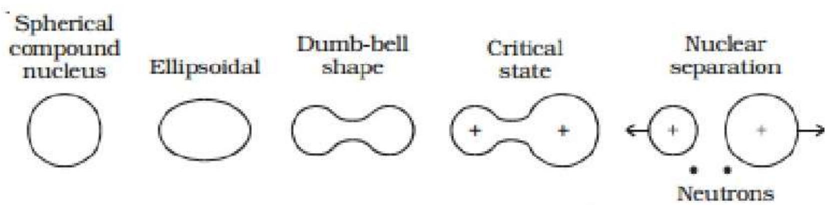


Fig Bohr – Wheeler's explanation of Nuclear fission

The various stages of deformation, leading to the final splitting of a liquid drop into two fragments are shown in Fig. For a light nucleus, surface tension dominates over electrostatic forces. Separation occurs only when the two fragments are connected by a narrow neck, requiring a critical energy equal to the difference between the original nucleus's energy and the total energy of the separated fragments. For a heavy nucleus, electrostatic forces are dominant. Even a small deformation increases repulsion, leading to early-stage separation.

The surface energy and Coulomb energy of an undeformed spherical nucleus are given by:

$$E_{So} = a_2 A^{\frac{2}{3}} = 4\pi r_o^2 S A^{\frac{2}{3}}$$

$$E_{Co} = \frac{a_3 Z^2}{A^{1/3}} = \frac{3ZA^2}{4\pi\varepsilon_o 5 r_o A^{1/3}}$$

Where, S is the surface energy per unit area and ro = 1.2fm. If Es and Ec represent the corresponding energies of the deformed nucleus, then the change in the combined surface and electrostatic energies due to deformation will be,

$$\Delta E = \Delta E_s - \Delta E_c = (E_s - E_{so}) + (E_c - E_{co})$$

Bohr and Wheeler, by straight forward calculation shown that,

$$\Delta E = \alpha_2^2 \left(\frac{2}{5} a_2 A^{2/3} - \frac{1}{5} a_3 \frac{Z^2}{A^{1/3}} \right)$$

For Z sufficiently large, ΔE will become negative which means that spontaneous fission will occur instantaneously. The limiting condition for this to happen is,

$$\frac{1}{5} a_3 \frac{Z^2}{A^{1/3}} > \frac{2}{5} a_2 A^{2/3}$$

Substituting the values of the parameters a_2 and a_3 in equation, we get

$$\left(\frac{Z^2}{A} \right)_{\text{lim}} \cong 50$$

So, nuclei with $Z^2/A > 50$ will be unstable against spontaneous fission. For the heaviest natural element, uranium, $Z^2/A = 36$, which is well below the limiting value, so that all naturally occurring nuclei are stable w.r.t. small deformation. Bohr and Wheeler on the basis of liquid drop model calculated the critical energy E_{crit} that must be supplied with the neutron. According to their calculation, $E_{\text{crit}} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/3}}$ MeV, Where, A is the atomic mass of the compound nucleus and Z is the atomic number. For, ${}_{92}^{236}U$ fission one gets $A = 236$ and $Z = 92$; $E_{\text{crit}} = 6.9$MeV.

Shell Model`

In 1948, M.G. Mayer provided convincing evidence for the existence of closed nuclear shells, leading to the development of the shell model. The following observations support the concept of magic numbers (2, 8, 20, 28, 50, 82, 126), which indicate enhanced nuclear stability:

Evidence Supporting the Shell Model

1. Binding Energy Peaks: The binding energy per nucleon increases significantly for nuclei with magic numbers, indicating exceptional stability.

2. Abundance of Stable Isotopes: Elements with Z = 20, 50, and 82 have more stable isotopes than neighbouring elements, showing enhanced nuclear stability at magic numbers.
3. Abundance of Stable Isotones: The number of stable isotones is significantly higher for N = 20, 50, and 82, supporting shell closures.
4. Natural Abundance Peaks: Analysis of nuclei in the Earth, Sun, and stars shows high relative abundances for nuclei with magic numbers, such as ^{16}O,^{40}Ca, ^{208}Pb.
5. Separation Energy Drop: The binding energy of an additional neutron drops suddenly after a magic number, confirming shell closure.
6. Spontaneous Neutron Emitters: Some isotopes undergo neutron emission, with the final products having neutron or proton numbers equal to 82 or 126.
7. Neutron Absorption Cross-Section: Nuclei with magic numbers have lower neutron absorption probabilities, indicating greater stability.
8. Discontinuities in Alpha Decay Energies: Alpha particle energies show sudden increases at N = 126, Z = 82, indicating strong nuclear binding.
9. Beta Decay Patterns: Similar discontinuities in beta decay rates suggest magic number stability.
10. Nuclear Shape and Quadrupole Moment: Nuclei with magic numbers are nearly spherical, confirming closed shell configurations, whereas deformed nuclei show non-zero quadrupole moments.

These findings collectively prove that nuclei with magic numbers correspond to closed shells, leading to enhanced nuclear stability, validating the nuclear shell model.

Basic Assumptions of the Shell Model

- Nucleons in a nucleus move independently in a common (mean) potential determined by the average motion of all the other nucleons.
- Protons and neutrons separately fill levels in the nucleus.
- Most of the nucleons are paired and a pair of nucleons contributes zero spin and zero magnetic moment. The paired nucleons thus form an inert core.
- The properties of odd A nuclei are characterized by the unpaired nucleon and odd–odd nuclei by the unpaired proton and neutron

Different forms of the potential have been used for the calculation of the nuclear energy levels, viz., the square well potential and the harmonic oscillator etc. None of these correspond to the actual potential which probably has a shape intermediate between the two as shown in fig. 2.

All the three different types of potential well candidates are given below.
Harmonic Oscillator

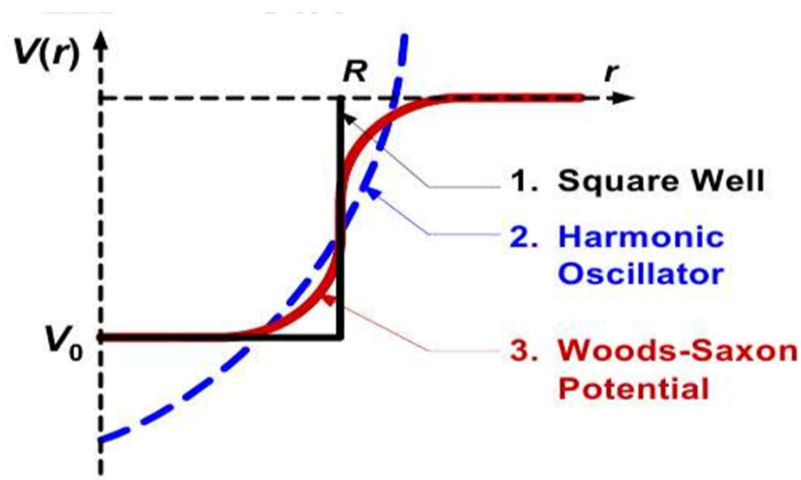
$$V(r) = \frac{1}{2} M\omega r^2$$

Square Well

$$\begin{aligned} V(r) &= -V_0 \qquad \text{for r} \leq R \\ &= +\infty \qquad \text{for r} > R \end{aligned}$$

Wood-Saxon Potential

$$V(r) = -\frac{V_0}{1 + \exp[(r-R)/a]}$$



Spin-Orbit Coupling

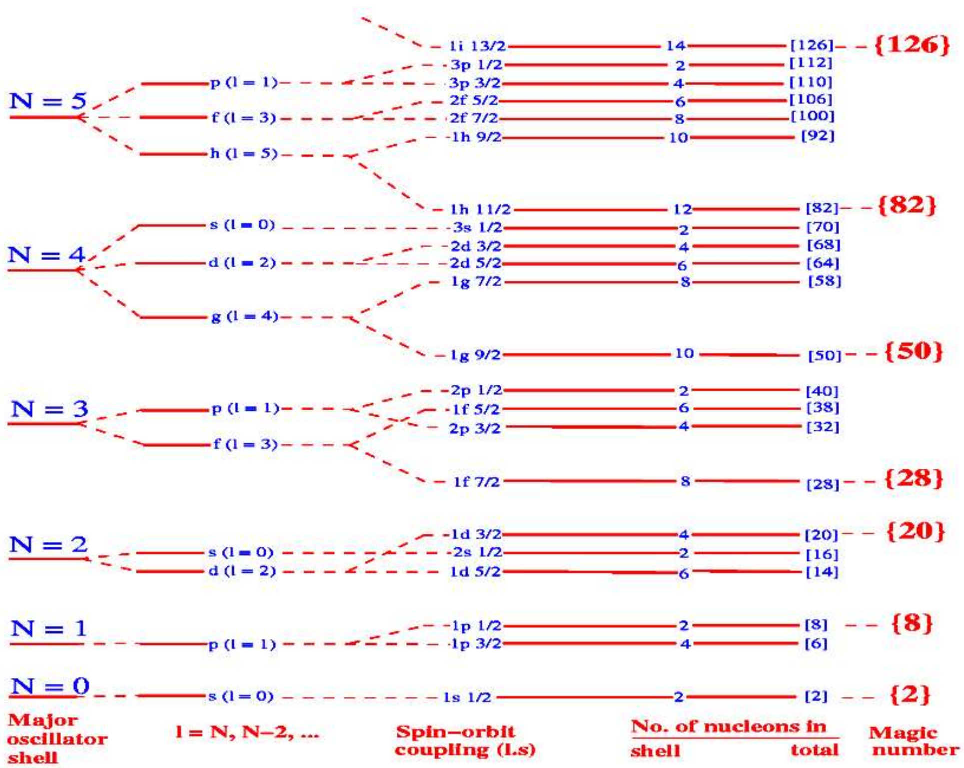
The results that a central force potential is able to account for the first three magic numbers only, 2, 8, 20, but not the remaining four, 28, 50, 82, 126. This situation does not change when other shapes of potential forms are used. For higher levels there are discrepancies in the magic numbers thus we need a more precise model to obtain a more accurate prediction. The implication is that something very fundamental about the single-particle interaction picture is missing in the description.

So we have to improve the previous potential forms in order to reproduce the magic numbers correctly. We should not want to make radical changes because this would destroy the physical content of this potential. Hence in order to predict the higher magic numbers, we need to take into account other interactions between the nucleons. The first interaction we analyze is the spin-orbit coupling.

In order to explain the disagreement at the higher magic numbers, M. G¨oppert Mayer, and independently D. Haxel J. Jensen and H. Suess in 1949 suggested to add a spin-orbit interaction term for each nucleon to the central potential *V(r)* which solved the problem of finding the magic numbers and gave the suitable separation between the shells.

The spin-orbit potential, which is non-central, can be written as

$$V_{ls}\,(\mathrm{r}) = -\,V_{0s}\frac{1}{r}\left(\frac{df}{dr}\right)\,\vec{l}.\vec{s}$$

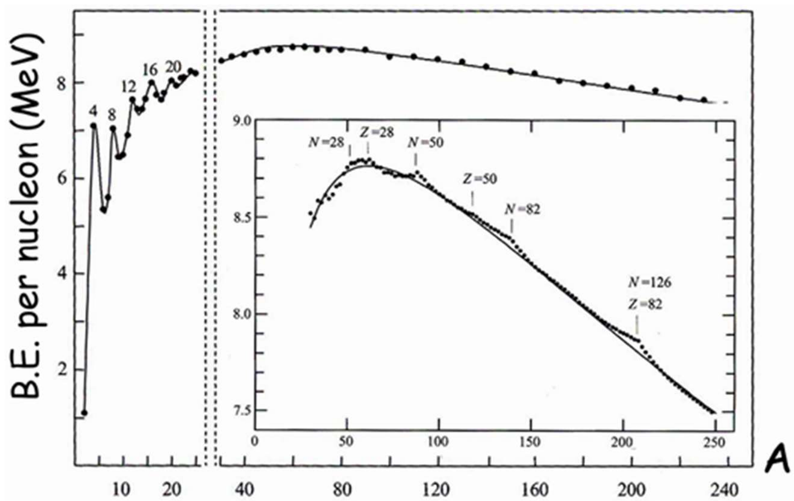


Magic Numbers

Magic numbers are specific numbers of protons or neutrons that result in exceptionally stable nuclear configurations. These numbers are:

2, 8, 20, 28, 50, 82, and 126

Nuclei with numbers of protons or neutrons values of 2, 8, 20, 28, 50, 82, and 126 are very



stable and show significant departures from the average nucleus behaviour. They represent the effects of the filled major shells analogous to the atomic shell model. The binding energy per nucleon is large for magic numbers.

Doubly magic nuclei extremely stable (where Z and N are magic)

• Energies in alpha and beta decay high when daughter nucleus is magic

• Nuclear radius is not changed much with Z, N at magic numbers

• 1st excited states for magic numbers higher than neighbours

• Spontaneous neutron emitters have magic number +1

• Terrestrial nuclear abundances for Z or N magic are greater than those for non

magic elements.

• Elements with Z/N magic have many more isotopes than with Z/N non-magic

Angular Momenta and Parity Of Ground States

Nuclear states have an intrinsic spin and a well defined parity, $\eta = \pm 1$, defined by the behaviour of the wavefunction for all the nucleons under reversal of their coordinates with the centre of the nucleus at the origin.

$$\Psi(-\mathbf{r_1},-\mathbf{r_2}\cdots -\mathbf{r_A}) = \eta\Psi(\mathbf{r_1},\mathbf{r_2}\cdots\mathbf{r_A}) .$$

The spin and parity of nuclear ground states can usually be determined from the shell model. Protons and neutrons tend to pair up so that the spin of each pair is zero and each pair has even parity ($\eta = 1$). Thus we have

• Even-even nuclides (both Z and A even) have zero intrinsic spin and even parity.

• Odd A nuclei have one unpaired nucleon. The spin of the nucleus is equal to the j value of that unpaired nucleon and the parity is $(-1)^l$, where l is the orbital angular momentum of the unpaired nucleon. For example $_{22}Ti^{47}$ (titanium) has an even number of protons and 25 neutrons. 20 of the neutrons fill the shells up to magic number 20 and there are 5 in the $1f_{7/2}$ state (l = 3, j = 7/2) Four of these form pairs and the remaining one leads to a nuclear spin 2 and parity $(-1)^3 = -1$.

• Odd-odd nuclei. In this case there is an unpaired proton whose total angular momen tum is j_1 and an unpaired neutron whose total angular momentum is j_2. The total spin of the nucleus is the (vector) sum of these angular momenta and can take val ues between $|j_1 - j_2|$ and $|j_1 + j_2|$ (in unit steps). The parity is given by $(-1)^{(l1+l2),}$ where l_1 and l_2 are the orbital angular momenta of the unpaired proton and neutron respectively.

For example $_3Li^6$ (lithium) has 3 neutrons and 3 protons. The first two of each fill the 1s level and the third is in the $1p_{3/2}$ level. The orbital angular mometum of each is l = 1 so the parity is $(-1)\times(-1) = +1$ (even), but the spin can be anywhere between 0 and 3.

Magnetic Moment

Since nuclei with an odd number of protons and/or neutrons have intrinsic spin they also in general possess a magnetic dipole moment. The unit of magnetic dipole moment for a nucleus is the “nuclear magneton” defined as

$$\mu_N = \frac{e\hbar}{2m_p}$$

which is analogous to the Bohr magneton but with the electron mass replaced by the proton mass. It is defined such that the magnetic moment due to a proton with orbital angular momentum **l** is μ_N **l**.

Experimentally it is found that the magnetic moment of the proton (due to its spin) is

$$\mu_p = 2.79\mu_N = 5.58\mu_N s, \left(s = \frac{1}{2}\right)$$

and that of the neutron is

$$\mu_n = -1.91\mu_N = -3.82\mu_N s, \left(s = \frac{1}{2}\right)$$

If we apply a magnetic field in the z-direction to a nucleus then the unpaired proton with orbital angular momentum **l**, spin **s** and total angular momentum **j** will give a contribution to the z− component of the magnetic moment

$$\mu^z = (5.58s^z + l^z)\mu_N$$

As in the case of the Zeeman effect, the vector model may be used to express this as

$$\mu^z = \frac{(5.58 < \mathbf{s} \cdot \mathbf{j} > + < \mathbf{l} \cdot \mathbf{j} >)}{< \mathbf{j}^2 >} j^z \mu_N$$

using

$$< \mathbf{j}^2 > = j(j+1)\hbar^2$$

$$< \mathbf{s} \cdot \mathbf{j} > = \frac{1}{2}(< \mathbf{j}^2 > + < \mathbf{s}^2 > - < \mathbf{l}^2 >)$$

$$= \frac{\hbar^2}{2}(j(j+1) + s(s+1) - l(l+1))$$

$$< \mathbf{l} \cdot \mathbf{j} > = \frac{1}{2}(< \mathbf{j}^2 > + < \mathbf{l}^2 > - < \mathbf{s}^2 >)$$

$$= \frac{\hbar^2}{2}(j(j+1) + l(l+1) - s(s+1))$$

We end up with expression for the contribution to the magnetic moment

$$\mu = \frac{5.58(j(j+1) + s(s+1) - l(l+1)) + (j(j+1) + l(l+1) - s(s+1))}{2j(j+1)} j\mu_N$$

and for a neutron with orbital angular momentum l' and total angular momentum j' we get (not contribution from the orbital angular momentum because the neutron is uncharged)

$$\mu = -\frac{3.82\big(j'(j'+1) + s(s+1) - l'(l'+1)\big)}{2j'(j'+1)} j'\mu_N$$

Thus, for example if we consider the nuclide $^{7}_{3}$Li for which there is an unpaired proton in the $2p_{\frac{3}{2}}$ state $\left(l = 1, j = \frac{3}{2}\right.$ then the estimate of the magnetic moment is

$$\mu = \frac{5.58\left(\frac{3}{2}\times\frac{5}{2}+\frac{1}{2}\times\frac{3}{2}-1\times 2\right)+\left(\frac{3}{2}\times\frac{5}{2}+1\times 2-\frac{1}{2}\times\frac{3}{2}\right)}{2\times\frac{3}{2}\times\frac{5}{2}}\frac{3}{2} = 3.79\mu_N$$

The measured value is $3.26\mu_N$ so the estimate is not too good. For heavier nuclei the estimate from the shell model gets much worse.

The precise origin of the magnetic dipole moment is not understood, but in general they cannot be predicted from the shell model. For example for the nuclide $^{17}_{9}$ F (fluorine), the measured value of the magnetic moment is $4.72\mu_N$ whereas the value predicted form the above model is $-0.26\mu_N$. There are contributions to the magnetic moments from the nuclear potential that is not well-understood.

Schmidt model

The Schmidt model is a theoretical model used in nuclear physics to estimate the magnetic dipole moments and electric quadrupole moments of atomic nuclei based on the single-particle shell model. It assumes that the nuclear magnetism primarily arises from the contribution of a single unpaired nucleon (proton or neutron) in an orbital around an inert core.

Single-Particle Approximation:

- The model considers only the unpaired nucleon in the valence shell, ignoring interactions between nucleons.

Magnetic Dipole Moment (μ):

- Calculated using the sum of the nucleon's intrinsic spin and orbital magnetic moments.
- Expressed as:

$$\mu = g_l l + g_s s$$

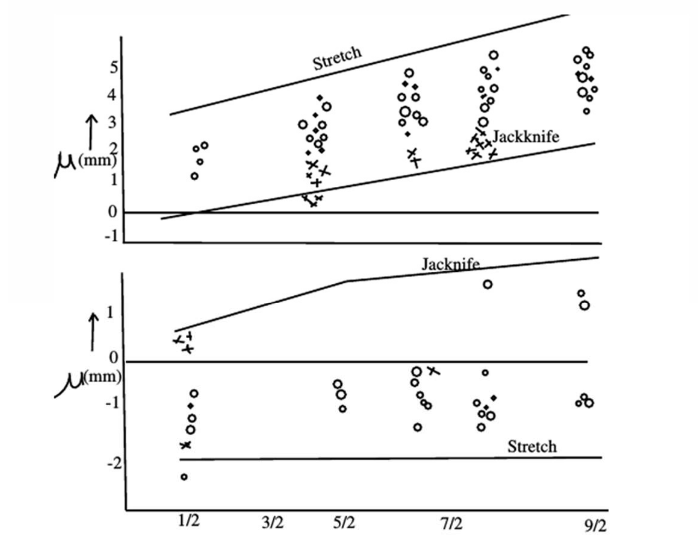


Figure: Magnetic dipole moments against angular momentum- Schmidt lines

where g_l and g_s are the orbital and spin g-factors.

Hence $\mu =$ sum of the components of the vectors $g_l l$ and $g_s s$ along the j. By applying the cosine rule to the triangle formed by the l, s and j, the above relation can be written as

$$\mu = g_l\sqrt{[l(l+1)]}\frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{ll(l+1)j(j+1)]}+g_s\sqrt{[s(s+1)]}}\frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[s(s+1)j(j+1)]}}$$

$$= \frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{[(j(j+1)]}}g_l + \frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[j(j+1)]}}g_s$$

Since for a single particle, the spin$s = 1/2$ and there are two possible cases. l parallel to s (Stretch case) ; $\mathrm{J} = 1 + s = 1 + 1/2$. l antiparallel to s (Jacknife case) ; $\mathrm{J} = 1 - s = 1 - 1/2$.

$$\mu = \left(J - \frac{1}{2}\right)g_l + \frac{1}{2}g_s \qquad \text{for stretch case ----------(1)}$$

$$\mu = \frac{J}{J+1}\left[\left(J + \frac{3}{2}\right)g_l - \frac{1}{2}g_s\right] \qquad \text{for Jackknife case------(2)}$$

These relations define two curves, for μ versus J , with the values $\mathrm{J} = 1 \pm 1/2$, for each class of odd even nucleus. The values of μ are known as the Schmidt value and the curves are known as Schmidt lines. When we substitute the above equations (2) and (3) the g factors which correspond to single nucleons are $g_l = 1$ and $g_s = 5.58$ for protons and $g_l = 0$ and $g_s = -3.82$ for neutrons.

Electric Quadrupole Moment

The Electric quadrupole moment **(Q)** is a property that describes the shape of a nucleus beyond its spherical symmetry. It arises when a nucleus has a non-spherical charge distribution, indicating nuclear deformation. Neutrons have no charge, so do not induce quadrupole moment. The electric quadrupole moment Q of a nucleus is the average of the quantity $(3z^2 - r^2)$ for the charge distribution in the nucleus. For a spherically symmetric charge distribution this average is zero and hence Q = 0 for even-even nuclei which have ground state spin $I = 0$ in the nucleus.

Significance of Q:

Q>0: Prolate shape (elongated along an axis, like a rugby ball).

Q<0: Oblate shape (flattened, like a discus).

Q=0: Spherical nucleus (no deformation).

$$Q=\int\rho(r)(3z^2-r^2)dV$$

where:

- ρ(r) is the charge density distribution,
- r is the radial distance from the center of the nucleus,
- z is the coordinate along the chosen symmetry axis,
- dV is the volume element.

In case of a nucleus with single unpaired proton the quadrupole moment is given as

$$Q = \langle r^2 \rangle$$

$$\langle r^r \rangle = \frac{3}{5}R^2 = \frac{3}{5}R_0^2A^{2/3}$$

⟨ r^r ⟩ is the mean square radius of the charge distribution which in the present case is equal to the mean square distance of the proton from the nuclear centre.

So, quadrupole moments give

$$\langle Q_{sp} \rangle = -\frac{2j-1}{2(j+1)}\langle r \rangle^2$$

The negative sign indicates that orbital motion of the proton in the equatorial plane makes the charge distribution an oblate spheroid. On the other hand an odd hole in the case of j would make the charge distribution a prolate spheroid for which Q>0. Thus both positive and negative values of Q are expected.

Ideally, a nucleus with a single odd-neutron should have no quadrupole moment. However, in actual the neutrons in nucleus interact with the nucleons of core to polarize it and generate a small quadrupole moment. The value of quadrupole moment for neutron is much smaller than the value for proton as indicated in figure.

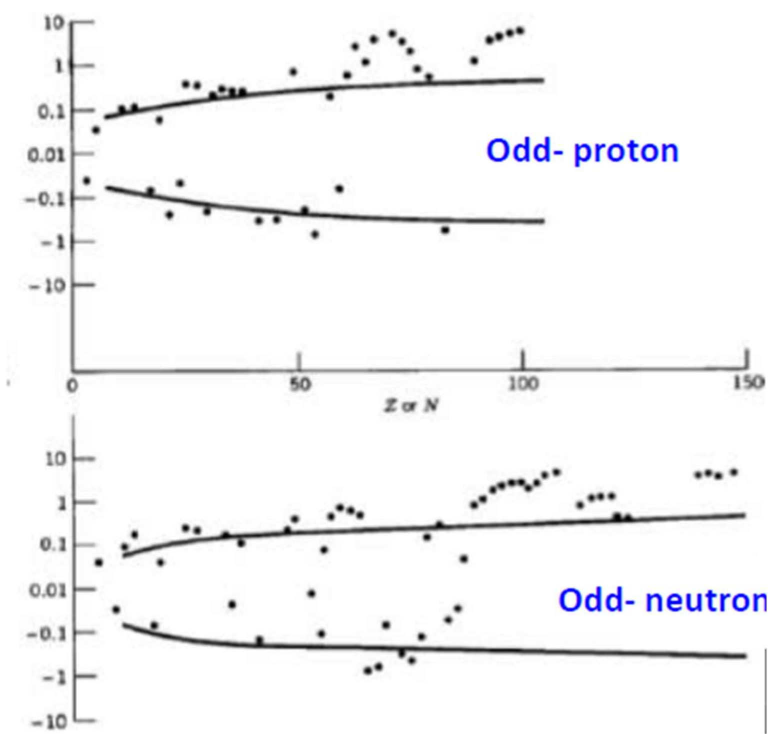


Figure: Variation of quadrupole moment for odd proton and odd neutron case

Bohr and Mottelson collective model

The collective model of the nucleus, developed by Aage Bohr and Ben Mottelson, describes nuclear motion by considering collective behavior of nucleons, rather than treating them as independent particles. This model successfully explains nuclear deformations, rotational spectra, and vibrational excitations**.**

The shell model is based on the assumption of the existence of a spherically symmetric potential in the nucleus, plus a spin-orbit coupling term. The different types of coupling of the angular momenta assumed for the loose nucleons outside the core given rise to the different forms of the shell model.

The shell model, with some refinements, has been successfully applied to explain many features of the nucleus in the ground state and in some of the excited states. However, it fails in explaining the observed large electric quadrupole moments (Q) of the nuclei in many cases and the quadrupole transition. In such cases where Q is n times the single particle value, we must assume that 2n particles are involved in producing the observed Q since the neutrons cannot directly contribute to Q. It is the collective motion of fairly large number of nucleons which determines the large values of Q for nuclei far from closed shells. To explain these failures of the shell model by introducing the idea of deformation in the shape of the nuclear core due to the motion of the loose odd nucleon outside the core in odd A nuclei. Such deformation would cause the quadruple moment to be higher than the single particle value. Transition rate is also increased. Further elaborated the model, combining the single particle and collective motions into a unified model which gave a more complete description of the deformed nuclei.

In nearly spherical nuclei, the coupling between the collective motion of the nucleons in the core and the motion of the loose nucleons outside the core is weak. On the other hand, for strong coupling, the surface is distorted and the potential felt by the loose particles is not spherically symmetric. These particles, moving in a non-spherically symmetric shell model potential, maintains the deformed nuclear shape. The situation is similar to that in a linear molecule, we can then write the total energy as the sum of the rotational, vibrational and nucleonic energies of the nucleus, as in the case of the molecule.

In the present case, the nucleonic energy replaces the electronic energy of the molecules.

$$E_{tot} = E_{rot} + E_{vib} + E_{nuc}$$

The collective motion of the nuclear core gives rise to the rotational and vibrational term, while nucleonic energy term is due to the motion of the loose nucleons. Mathematically this means that Etot is composed of three additive parts containing rotational energy state, vibrational energy state and nucleonic energy state.

Rotational And Vibrational Bands

Rotational Bands

Rotational bands are a characteristic feature of deformed nuclei, particularly in the rare-earth and actinide regions, where the nucleus exhibits an ellipsoidal shape. These bands arise due to the collective rotation of the entire nucleus.

Energy Levels in Rotational Bands

For a non-spherical (axially deformed) nucleus, the energy levels follow a rigid rotor model, given by:

$$E(J) = \frac{\hbar^2}{2I} J(J+1)$$

where:

- J is the total angular momentum,
- I is the moment of inertia,
- $\hbar$ is the reduced Planck's constant.

Key Features of Rotational Bands

- Even-even nuclei**:** The ground state has $J^\pi=0^+$ and rotational levels increase as $2^+,4^+,6^+$, etc.
- Energy spacing between consecutive levels decreases with increasing J due to increasing moment of inertia.
- E2 transitions (quadrupole transitions) dominate in rotational bands, following selection rules $\Delta J=\pm 2$

Vibrational Bands

Vibrational bands are found **in** spherical or near-spherical nuclei and correspond to quantum excitations of nuclear shape oscillations.

Types of Vibrations

1. Monopole (Breathing Mode) - Compression and expansion of the nucleus (less common).
2. Quadrupole Vibrations - Most prominent, leading to:

- Beta (β) vibrations: Oscillations in nuclear elongation (symmetric deformation).
- Gamma (γ) vibrations: Oscillations in nuclear asymmetry (shape tilting).

3. Octupole Vibrations - Associated with pear-shaped deformations.

Energy Levels in Vibrational Bands For quadrupole vibrations (the most common case):

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

where:

- n is the vibrational quantum number,
- ω is the frequency of oscillation.

The first vibrational state typically appears around $1 - 3\text{MeV}$ above the ground state. Key Features of Vibrational Bands

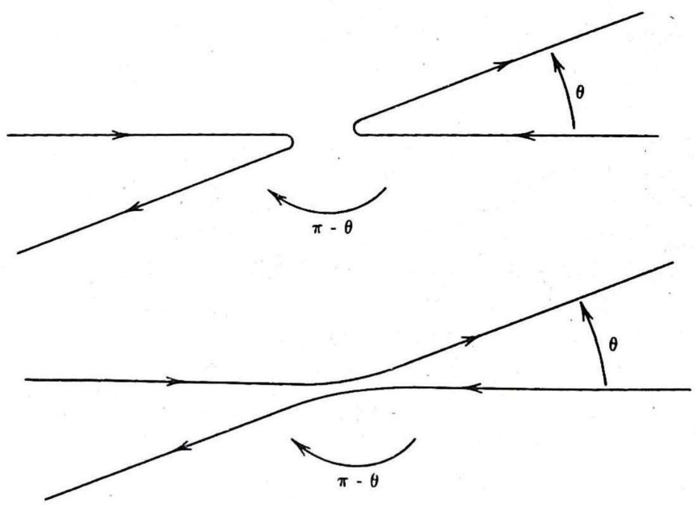
- First excited state in spherical nuclei is usually a quadrupole vibration with $J^\pi - 2^+$.
- Higher vibrational states include $0^+, 3^-, 4^+$, etc.
- E2 transitions dominate, with selection rules $\Delta J = 0, \pm 2$.

UNIT II: NUCLEAR FORCES

Nucleon – nucleon interaction – Tensor forces – properties of nuclear forces – ground state of deuteron – Exchange Forces - Meson theory of nuclear forces – Yukawa potential – nucleon-nucleon scattering –effective range theory – spin dependence of nuclear forces – charge independence and charge symmetry – isospin formalism.

Nucleon - nucleon interactions

There is one very important difference between the scattering of identical nucleons (proton-proton and neutron-neutron scattering) and the scattering of different nucleons (neutron-proton scattering). Nucleons have spin $\frac{1}{2}$, their wave functions must be antisymmetric with respect to interchange of the nucleons. If we again consider only low-energy scattering, so that $\ell = 0$, interchanging the spatial coordinates of the two particles gives no change in sign. Thus the wave function is symmetric with respect to interchange of spatial coordinates and must therefore be antisymmetric with respect to interchange of spin coordinates in order that the total (spatial times spin) wave function be antisymmetric.

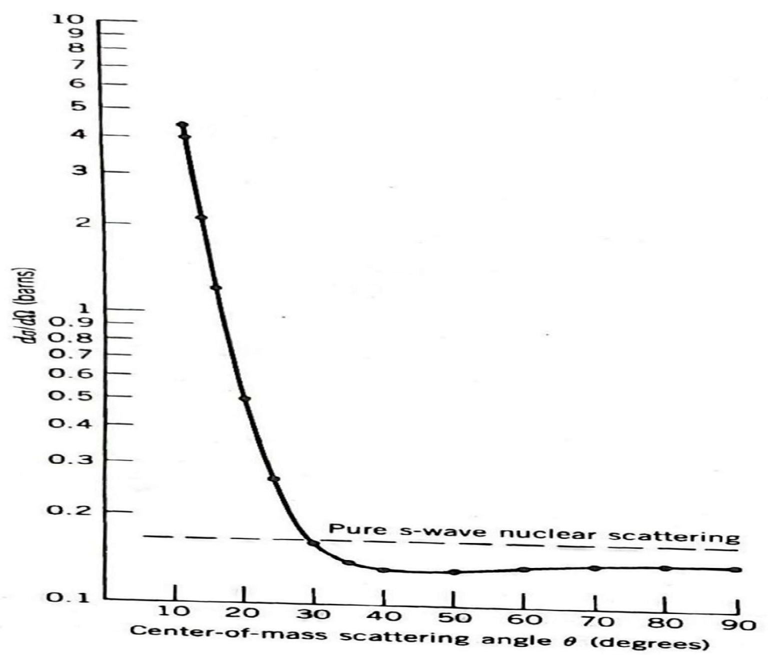


Scattering of identical particles in the center-of-mass system.

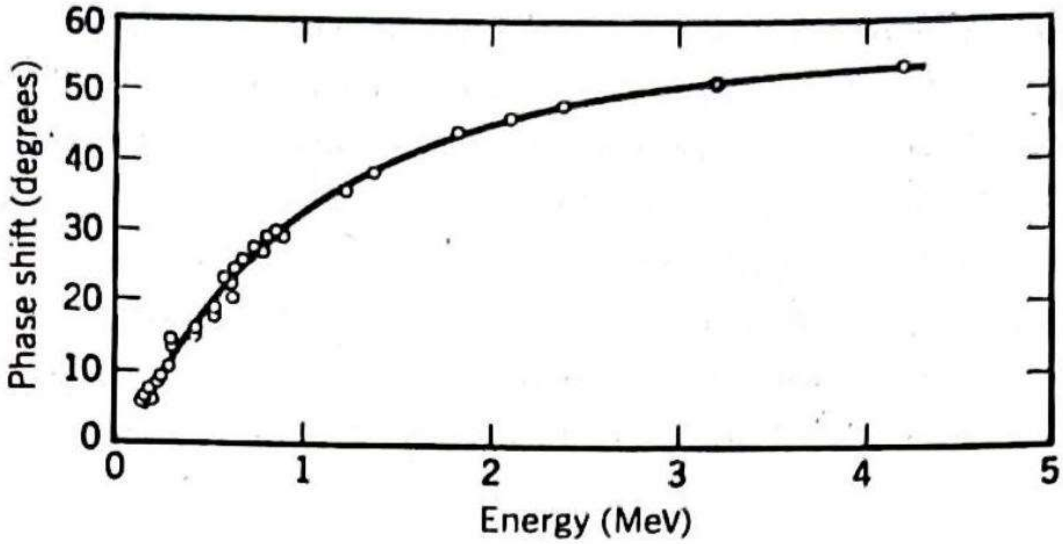
Let's first consider scattering between two protons; the, wave function must describe both Coulomb and nuclear scattering, and there will be an additional Coulomb-nuclear interference term in the cross section. (The scattered wave function must include one term resulting from Coulomb scattering and another resulting from nuclear scattering; the Coulomb term must vanish in the limit $e \to 0$, and the nuclear term must vanish as the nuclear potential vanishes, in which case $\delta_0 \to 0$. The differential cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{4T^2}\left\{\frac{1}{\sin^4(\theta/2)}+\frac{1}{\cos^4(\theta/2)}-\frac{\cos[\eta\ln\tan^2(\theta/2)]}{\sin^2(\theta/2)\cos^2(\theta/2)}\right.$$
$$\left.-\frac{2}{\eta}(\sin\delta_0)\left(\frac{\cos[\delta_0+\eta\ln\sin^2(\theta/2)]}{\sin^2(\theta/2)}+\frac{\cos[\delta_0+\eta\ln\cos^2(\theta/2)]}{\cos^2(\theta/2)}\right)\right.$$

Here T is the laboratory kinetic energy of the incident proton (assuming the target proton to be at rest), θ is the scattering angle in the center-of-mass system, δ_0 the $\ell=0$ phase shift for pure nuclear scattering, and $\eta=(e^2/4\pi\epsilon_0\hbar c)\beta^{-1}=\alpha/\beta$, where α is the fine-structure constant (with a value of nearly $\frac{1}{137}$) and $\beta=v/c$ is the (dimensionless). The $\sin^{-4}(\theta/2)$ is characteristic of Coulomb scattering, also known as Rutherford scattering. Since the two protons are identical, we cannot tell the case in which the incident proton comes out at θ and the target proton at $\pi-\theta$ (in the center-of-mass system) from the case in which the incident proton comes out at $\pi-\theta$ and the target proton at θ. Thus, the scattering cross section must include a characteristic Coulomb (Rutherford) term $\sin^{-4}(\pi-\theta)/2=\cos^{-4}(\theta/2)$. This term describes the interference between Coulomb scattering at θ and at $\pi-\theta$. These two terms result from the interference between Coulomb and nuclear scattering. The last term is the pure nuclear scattering term. In the limit $e\to 0$ (pure nuclear scattering)



The cross section for low-energy proton-proton scattering at an incident proton energy of 3.037 MeV. Fitting the data points to Equation 4.43 gives the s-wave phase shift $\delta_0 = 50.966°$. The cross section for pure nuclear scattering would be 0.165. The observation of values of the cross section smaller than the pure nuclear value is evidence of the interference



between the Coulomb and nuclear parts of the wave function. The next step in the interpretation of these data is to represent the scattering in terms of energy-independent quantities such as the scattering length and effective range Unfortunately, this cannot easily be done because the Coulomb interaction has infinite range and even in the $k \to 0$. The s-wave phase shift for pp scattering as deduced from the experimental results of several workers. obtain values for the proton-proton scattering length and effective range:

$$\dot{a} = -7.82 \pm 0.01\text{fm}$$
$$r_0 = 2.79 \pm 0.02\text{fm}$$

The effective range is entirely consistent with the singlet np values deduced in the previous section. The scattering length, which measures the strength of the interaction, includes Coulomb as well as nuclear effects and thus cannot be compared directly with the corresponding np value.

Tensor force

The nucleon-nucleon force has a noncentral or tensor component. This part of the force does not conserve orbital angular momentum, which is a constant of the motion under central forces. Tensor force implies force dependent on spin.

Non-Central Force:

From the small deviation of deuteron wavefunction from spherical symmetry, we can calculate that non-central force is also very small compared to the central force. The tensor force resembles a magnetic interaction between two dipoles. It results in the lower potential energy for n and p when the line joining them is parallel to spin direction than when it is perpendicular.

General form:

Let us now see whether we can construct a potential which depends not only on relative position vector of particles r, but also on their spin orientation σ1 and σ2 and which can account for the quadrupole moment. This potential must be invariant under rotations and reflections of coordinate system which we use to describe the relative motion of particles and hence must be scalar. Wigner and Eisenbud in1941 has showed that if interactions are assumed to be invariant with respect to displacement, rotation and inversion of observer's co- ordinate system and independent of particle velocity, general potential can be written as

$$V(r) = V_1(r) + V_2(r)\sigma_1.\sigma_2 + V_3(r)S_{12}$$

Where the potentials V may depend on orbital momentum of two particle system as well as on charge of particles. The σ's being particle spin axial vectors. The reason to this limited choice are:

1. The vector r changes to -r under the reflection and thus must occur in even powers only.
2. Derivatives of r cannot occur since forces are velocity independent.
3. Since σ1 and σ2 are not invariant against rotation and σ1 . σ2 is invariant.
4. σ1.r is invariant against rotation, but not against inversion, since r→-r and σ → -σ on inversion. Because of this, only even power of (σ.r) may occur such as (σ1.r)(σ2.r).

The first term in the eqn is corresponding to the central force, second term corresponding to spin dependant but central force. The value of σ1. σ2 is 1 for triplet state and -3 for singlet state. The last term is corresponding to the interaction potential which not only depend on separation between neutron and proton but also on angle which make their spin make with line joining two particles.

The non-central character of interaction is contained in tensor operator called tensor force operator and is given by

$$S_{12} = 3(\frac{(\sigma_1 . r)(\sigma_2 . r)}{r^2} - \sigma_1 \sigma_2)$$

First term gives dependence of interaction upon the angle between the particle spin particle vector r. The second term has been subtracted so that the average of S12 over all directions r is zero.

Properties of nuclear force:

1. At short distances it is stronger than the Coulomb force; the nuclear force can overcome the Coulomb repulsion of protons in the nucleus.
2. At long distances, of the order of atomic sizes, the nuclear force is negligibly feeble; the interactions among nuclei in a molecule can be understood based only on the Coulomb force.
3. Some particles are immune from the nuclear force; there is no evidence from atomic structure, for example, that electrons feel the nuclear force at all.
4. The nucleon-nucleon force seems to be nearly independent of whether the nucleons are neutrons or protons. This property is called ***charge independence.***
5. The nucleon-nucleon force depends on whether the spins of the nucleons are parallel or antiparallel.
6. The nucleon-nucleon force includes a repulsive term, which keeps the nucleons at a certain average separation.
7. The nucleon-nucleon force has a noncentral or ***tensor*** component. This part of the force does not conserve orbital angular momentum, which is a constant of the motion under central forces.

Ground state of deutron

The ground state of the deuteron: The deuteron ground state wave function in the presence of a tensor force can be written as

$$\psi = \psi_S + \psi_D = \frac{u(r)}{r}\chi_S + \frac{w(r)}{r}\chi_D$$

where χ_S and χ_D are functions of the angles describing the orientation of the neutron-proton separation vector r and of the spin variable of neutron and proton together. χ_S is independent of angles and symmetric in the neutron and proton spin, χ_D has a complicated dependence of ingles.

The normalized spin angle wave functions Y_{JLS}^M belonging to a state of total angular momentum J whose z-component is M and which is a combination of an orbital angular momentum L with a spin S are used. Thus χ_S and χ_D are replaced by Y_{101}^M and Y_{121}^M respectively. The probability of finding the deuteron in the ^3S state is given uy

$$P_S = \int_0^\infty u^2(r)dr$$

and the probability of finding the deuteron in the ^3D state is given by

$$P_D = \int_0^\infty w^2(r)dr$$

The radial functions are normalized so that

$$P_S + P_D = \int_0^\infty [u^2(r) + w^2(r)]dr = 1$$

Table 8.2 shows that the tensor force operator S_{12} acting on the $^3S_1$ wavefunction Y_{101} can only lead to a linear combination of the $^3S_1$ and $^3D_1$ wave functions, i.e.,

$$S_{12}Y_{101}^M = aY_{101}^M + bY_{121}^M = bY_{121}^M$$

Here $a = 0$, because Y_{101}^M is independent of the direction ($l = 0$) and the operation can not result in a spherical symmetric state $l = 0$.

$$S_{12}Y_{121}^M = bY_{101}^M + cY_{121}^M$$

Here the constant b is same as in equation (148) because the tensor operator S_{12} is Hemitean. The calculations give $b = \sqrt{8}$ and $c = -2$.

Outside the range of nuclear forces, the deuteron ground state S-wave function, follows from the second order differential equation (Schrodinger equation) is given by

$$u(r) = N_S e^{-ar}$$

and the D-wave function

$$w(r) = N_D e^{-\alpha r}\left[1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2}\right]$$

Here $1/\alpha$ is the size of the deuteron and N_S and N_D are the normalization constants. A rough estimate of N_S can be obtained by neglecting D state probability $(p_S \simeq 1)$, as

$$\int_0^\infty u^2(r)dr \simeq 1 \text{ or } N_S^2/2\alpha = 1$$

The value of N_D depends on the strength of the tensor force, will be determined later. Magnetic moment of deuteron. We know that the small discrepancy between the sum of the magnetic moment of proton and neutron and the measured value for the deuteron can be interpreted as a contribution of the orbital motion of the proton in the D-state in the deuteron ground state. If nuclear forces are supposed to be central forces, the difference will be zero. This contribution can appear only with the non-central forces. The operator describing the magnetic moment of deuteron is

$$\mu = \mu_p \sigma_p + \mu_n \sigma_n + l_p,$$

where μ_n and μ_p are the magnetic moments of neutron and proton measured in nuclear magnetons, σ_n and σ_p their unitary spin operators and l_p is the orbital angular momentum of the proton about the center of mass of the bound system. The uncharged neutron cannot contribute any magnetic moment by orbital motion alone. In the centre of mass system the orbital angular momentum of the proton is half of the combined orbital angular momentum L.

Since

$$\mu_n \sigma_n + \mu_p \sigma_p = \frac{1}{2}(\mu_n + \mu_p)(\sigma_n + \sigma_p) + \frac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_p)$$

Therefore, equation can be written as

$$\mu = \mu_p \sigma_p + \mu_n \sigma_n + l_p$$

where μ_n and μ_p are the magnetic moments of neutron and proton measured in nuclear magnetons, σ_n and σ_p their unitary spin operators and $\boldsymbol{l_p}$ is the orbital angular momentum of the proton about the center of mass of the bound system. The uncharged neutron cannot contribute any magnetic moment by orbital motion alone. In the centre of mass system the orbital angular momentum of the proton is half of the combined orbital angular momentum L.

Since

$$\mu_n\sigma_n + \mu_p\sigma_p = \frac{1}{2}(\mu_n + \mu_p)(\sigma_n + \sigma_p) + \frac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_p)$$

Therefore equation (153) can be written as

$$\boldsymbol{\mu} = \frac{1}{2}(\mu_n + \mu_p)\frac{1}{2}(\sigma_n + \sigma_p) + \frac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_n) + \frac{1}{\boldsymbol{L}}\boldsymbol{L}$$

Since the operator $(\sigma_n - \sigma_p)$ in the second term vanishes for a triplet state (as spins a 207/461 d $= \mathbf{L} + \mathbf{S} = \mathbf{L} + \frac{1}{2}(\sigma_n + \sigma_p)$, hence the magnetic moment operator becomes

$$\mu = (\mu_n + \mu_p)\mathbf{I} - \left(\mu_n + \mu_p - \frac{1}{2}\right)\mathbf{L}$$

The observed value of μ is the expectation value of this expression in the state with $I_z = I$. We therefore, can replace $\mathbf{L}$ by L_{z^*}. With the usual theory of quantum operators, we may write

$$\mathbf{L} \to L_z = \frac{\mathbf{L}\cdot\mathbf{I}}{I^2}I_z = \frac{I(I+1) + L(L+1) - S(S+1)}{2I(I+1)}I_z$$

Since

$$\mu_n\sigma_n + \mu_p\sigma_p = \frac{1}{2}(\mu_n + \mu_p)(\sigma_n + \sigma_p) + \frac{1}{2}(\mu_n - \mu_p)(\sigma_n - \sigma_p)$$

Since the operator $(\sigma_n - \sigma_p)$ in the second term vanishes for a triplet state (as spins are parallel) and I $= \mathbf{L} + \mathbf{S} = \mathbf{L} + \frac{1}{2}(\sigma_n + \sigma_p)$, hence the magnetic moment operator becomes

$$\mu = (\mu_n + \mu_p)\mathbf{I} - \left(\mu_n + \mu_p - \frac{1}{2}\right)\mathbf{L}$$

The observed value of μ is the expectation value of this expression in the state with $I_z = I$. We therefore, can replace $\mathbf{L}$ by L_z. With the usual theory of quantum operators, we may write

$$\therefore\ \mathbf{L} \to L_z = \frac{\mathbf{L}\cdot\mathbf{I}}{I^2}I_z = \frac{I(I+1) + L(L+1) - S(S+1)}{2I(I+1)}I_z$$

For the deuteron, $I(I+1) = S(\text{ S}+1) = 2$ and in a mixture of S and D states, $L(L+1)$ is 0 for the D-sate and is 6 for the D-state. Therefore $[L(L+1)]_{av} = 0 \times p_S + 6 \times p_D = 6p_D$. Here

p_D and p_S represent the D-and S state probabilities respectively. For the deuteron $I = 1$, the value of I_z in the state with $I_z = I$ is unity. Thus

the expectation value of $\mathbf{L_z}$ is obtained by using eqn.

$$\langle\psi|L|\psi\rangle = 0 \text{ for the } S\text{-state}$$
$$= 3/2 \text{ for the } D\text{-state.}$$

The deuteron's magnetic moment in units of the nuclear magneton is given by

$$\mu_d = P_S\langle\psi_S|\mu|\psi_S\rangle + P_D < \psi_D|\mu|\psi_D\rangle$$

Substituting the values in the above equation, we get

Since $P_{.S} + P_D = 1$, therefore

$$\mu_d = P_S(\mu_n + \mu_p) + P_D\left[(\mu_n + \mu_p) - \frac{3}{2}\left(\mu_n + \mu_p - \frac{1}{2}\right)\right]$$

$$\mu_d = (\mu_n + \mu_p) - \frac{3}{2}P_D\left(\mu_n + \mu_p - \frac{1}{2}\right)$$

The last term gives a direct measure of the D-state probability P_D. The experimental results imply a D state probability of about 3.93%. The above relation does not give accurate results. There are various other causes which can give corrections, especially of relativistic effects. Hence the measured magnetic moment gives only a rough estimate of the D-state probability. One may expect that the D-state probability P_D lies somewhere between 2 and 8%. Due to this reason, the $n - p$ system is considered to be made up of 96%S-state and 4%D-state in the deuteron.

Exchange forces

We shall first investigate properties of different possible types of exchange forces between two nucleons.

The wave equation in the C-system can be written as

$$\frac{\hbar^2}{2\mu}\nabla^2\Psi + E\Psi = V\Psi$$

Here μ is the reduced mass. $\Psi = \Psi(r_1, r_2)\chi(\sigma_1, \sigma_2) = \psi_{12}\chi_{12}$ is a function of the position and spin co-ordinates of the two particles.The following possibilities may arise.

(i) Ordinary or Wigner force:

$V = V(r)P_W$ where $P_W = 1$ for all states. P_w is known as the Wigner operator. This would make the wave equation

$$\left(\frac{\hbar^2}{2\mu}\nabla^2 + E\right)\psi_{12}\chi_{12} = V(r)P_W\psi_{12}\chi_{12}$$
$$= V(r)\psi_{12}\chi_{12}$$

The interaction potential has the same sign in all states. Since it is attractive in the ^{3} S-state ($L = 0$) it must be attractive in all states. Such a force does not produce saturation.

(ii) Majorana space-exchange force:

$V = V(r)P_M$ where P_M is the Majorana or space exchange operator:

$$P_M\psi_{12} = P_M\psi(r_1, r_2) = \psi(r_2, r_1) = \psi_{21}$$

So the operator P_M has the effect of interchanging the space coordinates of the two particles. For the two nucleon system such interchange of the coordinates causes $r = r_1 - r_2$ to become $-r$ i.e., the effect of the operator P_M is equivalent to that of the parity operator which multiplies $\Psi_{12} = \psi(r_1, r_2)$ by $(-1)^L$, L being the orbital angular momentum. So the wave equation become

$$\left(\frac{\hbar^2}{2\mu}\nabla^2 + E\right)\psi_{12}\chi_{12} = V(r)P_M\psi_{12}\chi_{12} = V(r)\psi_{21}\chi_{12}$$
$$= (-1)^L V(r)\psi_{12}\chi_{12}$$
$$= +V(r)\psi_{12}\chi_{12} \text{ for } L = 0, \text{ even}$$
$$= -V(r)\psi_{12}\chi_{12} \text{ for } L \text{ odd.}$$

Since the neutron-proton interaction potential is attractive in the ^3S-state ($L = 0$), it must be attractive in all L-even states (D, G, etc., with $L = 2,4$ etc.,) and repulsive in all L-odd states (P, F etc., with $L = 1,3$, etc.). As stated above, such a force could provide saturation.

(iii) Bartlett or spin-exchange force:

We write $V = V(r)P_B$ where the spin exchange or Bartlett operator P_B has the property that

$$P_B\chi_{12} = P_B\chi(\sigma_1, \sigma_2) = \chi(\sigma_2, \sigma_1) = \chi_{21}$$

Thus P_B interchanges the spins of the two nucleons. For two particles of spin $1/2$ each, there are three symmetric spin states with $S = 1, S_z = 0, \pm 1$ and one antisymmetric spin state with $S = 0, S_z = 0$ (see § 17.12). Hence we can write in general

$$\chi_{21} = \chi(\sigma_2, \sigma_1) = (-1)^{S+1}\chi(\sigma_1, \sigma_2) = (-1)^{S+1}\chi_{12}$$

The wave function becomes

$$\begin{aligned}\left(\frac{\hbar^2}{2\mu}\nabla^2 + E\right)\Psi_{12}\chi_{12} &= V(r)P_B\Psi_{12}\chi_{12} \\ &= V(r)\Psi_{12}\chi_{21} \\ &= (-1)^{S+1}V(r)\psi_{12}\chi_{12} \\ &= +V(r)\psi_{12}\chi_{12} \text{ for } S = 1 \text{ (triplet)} \\ &= -V(r)\psi_{12}\dot{\chi}_{12} \text{ for } S = 0 \text{ (singlet).}\end{aligned}$$

Thus, all triplet state forces (^3P, ^3D etc.) are attractive since ^3S force is known to be attractive. On the other hand, all singlet state forces are repulsive (^1S, ^1P, ^1D etc.). However, n - p scattering experiments at low energies show that ^1S force is attractive, which contradicts the above conclusion. So, the Bartlett spin exchange force cannot be the only type of exchange force in the $n - p$ system.

(iv) Heisenberg space plus spin exchange force:

We write $V = V(r)P_H$ where the operator P_H has the property of interchanging both space and spin coordinates:

$$P_H\Psi_{12}\chi_{12} = P_H\psi(r_1, r_2)\chi(\sigma_1, \sigma_2)$$

$$\begin{aligned}&= \psi(r_2, r_1)\chi(\sigma_2, \sigma_1) \\ &= \psi_{21}\chi_{21} \\ &= (-1)^L\psi_{12} \cdot (-1)^{S+1}\chi_{12} \\ &= (-1)^{L+S+1}\psi_{12}\chi_{12}\end{aligned}$$

The wave equation becomes of

$$\left(\frac{\hbar^2}{2\mu}\nabla^2 + E\right)\Psi_{12}\chi_{12} = V(r)P_H\psi_{12}\chi_{12}$$

$$= (-1)^{L+S+1}V(r)\Psi_{12}\chi_{12}$$

$$= +V(r)\Psi_{12}\chi_{12} \text{ for } L = 0, \text{even}; S = 1$$

$$L = \text{odd}; S = 0$$

$$= -V(r)\Psi_{12}\chi_{12} \text{ for } L = 0, \text{even}; S = 0$$

$$L = \text{odd}; S = 1$$

So, the force is attractive in ^3S, ^3D etc., states as also in ^1P, ^1F etc., states. It is repulsive in iS, D etc., and also in ^3P, ^3F etc., states. This again contradicts the experimental observation, since ^3S and ^1S interactions must have the same sign (see above). Hence the Heisenberg space plus spin exchange force cannot also be the only type of exchange force in the $n - p$ system.

The estimated strengths of the ^3S and ^1S forces for the $n - p$ system can be explained by postulating a mixture of about 25% Heisenberg or Bartlett interaction with about 75% Wigner or Majorana interaction for a range of about 2.8 fm.

Meson theory of nuclear forces

In 1935, Hideki Yukawa proposed the Meson Theory of Nuclear Force to explain the strong interaction that binds protons and neutrons in the atomic nucleus. This theory suggests that nuclear forces arise due to the exchange of mesons, which act as force carriers between nucleons.

Key Concepts of Meson Theory

1. Yukawa Potential

Yukawa described the nuclear force using a mathematical expression known as the Yukawa potential:

$$V(r) = -g^2\frac{e^{-\mu}}{r}$$

where:

- g is the coupling constant,
- $\boldsymbol{\mu}$ is the meson mass (related to the range of the force),

- r is the separation distance between nucleons.

This potential describes how the force weakens rapidly over a short distance.

2. Range of Nuclear Force

- The nuclear force is short-ranged, acting only within 1-2 femtometers.
- Beyond this range, the force weakens exponentially, which explains why nucleons do not interact significantly at large distances.

3. Role of Mesons

- The pion (π-meson), discovered in 1947 by Cecil Powell, was confirmed as the mediator of nuclear forces.
- Other mesons, such as rho (ρ) and omega (ω) mesons, also contribute to nuclear interactions at shorter distances.
- Yukawa predicted the existence of a new particle with mass between an electron and a proton ($\sim 100 \mathrm{MeV}$).

Impact on Modern Nuclear Physics

- Explained why protons do not repel each other despite their positive charges.
- Provided a foundation for Quantum Chromodynamics (QCD), which describes nuclear interactions in terms of quarks and gluons.
- Helped develop modern nuclear models and effective field theories, such as chiral perturbation theory.
- Influenced experimental nuclear physics, including high-energy particle studies.

Yukawa potential

The Yukawa potential arises as the solution to the screened Poisson equation for a massive field, and it can be derived using Fourier transform techniques in quantum field theory. Starting with the Klein-Gordon Equation A massive scalar field ϕ satisfies the inhomogeneous Klein-Gordon equation in natural units ($\hbar = c = 1$):

$$(\nabla^2 - m^2)\phi(\mathbf{r}) = -\rho(\mathbf{r})$$

where $\rho(\mathbf{r})$ is a source term. For a point source at the origin:

$$\rho(\mathbf{r}) = g\delta^3(\mathbf{r})$$

where g is the coupling strength. Thus, the equation simplifies to:

$$(\nabla^2 - m^2)\phi(\mathbf{r}) = -g\delta^3(\mathbf{r})$$

Fourier Transform Taking the Fourier transform:

$$\tilde{\phi}(\mathbf{k}) = \int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r}}\phi(\mathbf{r})$$

Applying the Fourier transform to the differential equation:

$$(-k^2 - m^2)\tilde{\phi}(\mathbf{k}) = -g$$

Solving for $\tilde{\phi}(\mathbf{k})$:

$$\tilde{\phi}(\mathbf{k}) = \frac{g}{k^2 + m^2}$$

The potential $V(r)$ corresponds to $\phi(\mathbf{r})$, given by the inverse Fourier transform:

$$V(r) = \frac{1}{(2\pi)^3}\int d^3 k e^{i\mathbf{k}\cdot\mathbf{r}}\frac{g}{k^2 + m^2}$$

Switching to spherical coordinates ($d^3 k = 4\pi k^2 dk$), and integrating over angular variables:

$$V(r) = \frac{g}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{k^2 + m^2}\frac{\sin(kr)}{kr}dk$$

Using the standard result:

$$\int_0^\infty dk \frac{k^2 \sin(kr)}{(k^2 + m^2)kr} = \frac{\pi}{2}\frac{e^{-m}}{r}$$

we get:

$$V(r) = -\frac{g}{4\pi}\frac{e^{-m}}{r}$$

which is the Yukawa potential.

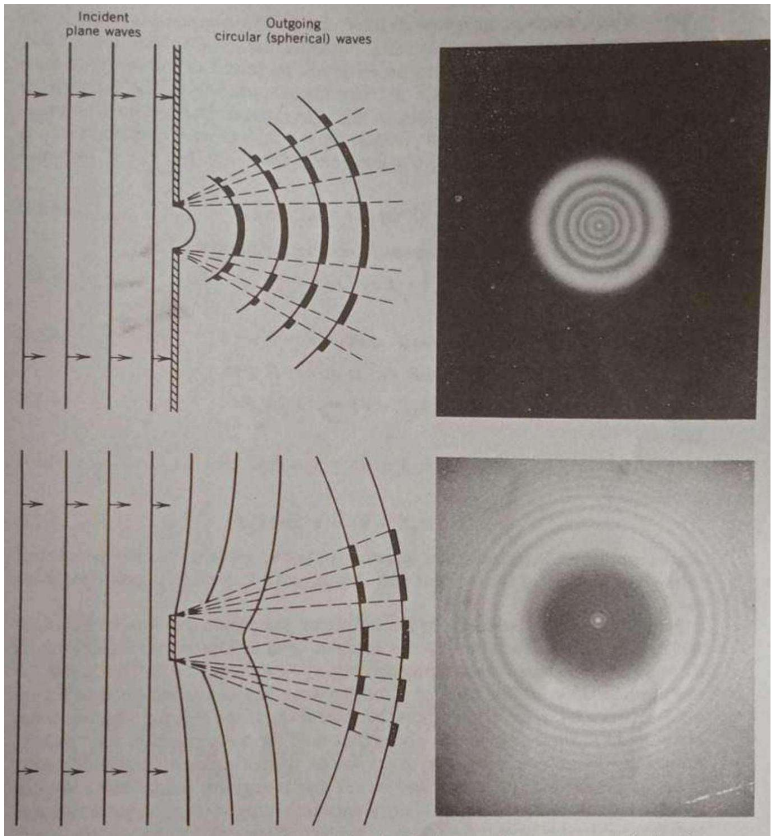
Nucleon - nucleon scattering

The study of the deuteron gives us a number of clues about the nucleon-nucleon interaction, the total amount of information available is limited. Because there are no excited states, we can only the deuteron: $\ell = 0$, parallel nucleon-nucleon interaction in states, if they were present, might have different / spins, $2 -$ fm separation. To study the nucleon-nucleon interaction in different values or spin orientation perform nucleon-nucleon scattering experiments.

There are three features of the optical diffraction that are analogous to the seattering of nucleons:

1. The incident wave is represented by a plane wave, while far from the obstacle the scattered wave fronts are spherical. The total energy content of any expanding spherical wave front cannot vary; thus its intensity (per unit area) must decrease like r^{-2} and its amplitude must decrease like r^{-1}.
2. Along the surface of any spherical scattered wave front, the diffraction is responsible for a variation in intensity of the radiation. The intensity thus depends on angular coordinates θ and ϕ.
3. A radiation detector placed at any point far from the obstacle would record both incident and scattered waves.

To solve the nucleon-nucleon scattering problem using quantum mechanics. we will again assume that we can represent the interaction by a square-well potential



Representation of scattering by (top) a small opening and (bottom) a small obstacle. The shading of the wavefronts shows regions of large and small intensity. On the right are shown photographs of diffraction by a circular opening and an opaque circular disk. Source of photographs: M. Cagnet, M. Francon, and J. C. Thrierr, Atlas of Optical Phenomena (Berlin: Springer-Verlag, 1962).

$$T = \frac{1}{2}mv^2 \ll \frac{\hbar^2}{2mR^2} = \frac{\hbar^2 c^2}{2mc^2R^2} = \frac{(200\text{MeV}\cdot\text{fm})^2}{2(1000\text{MeV})(1\text{fm})^2} = 20\text{MeV}$$

If the incident energy is far below 20 MeV.

The solution to the square-well problem for $r < R$ is given by Equation 4.3; as before, $B = 0$ in order that $u(r)/r$ remain finite for $r \to 0$. For $r > R$, the wave function is

$$u(r) = C'\sin k_2 r + D'\cos k_2 r$$

with $k_2 = \sqrt{2mE/\hbar^2}$. It is convenient to rewrite Equation 4.12 as

$$u(r) = C\sin(k_2 r + \delta)$$

Were,

$$C' = C\cos\delta \text{ and } D' = C\sin\delta$$

The boundary conditions on u and du/dr at $r = R$ give

$$C\sin(k_2R + \delta) = A\sin k_1R$$

and

$$k_2C\cos(k_2R + \delta) = k_1A\cos k_1R$$

Dividing,

$$k_2\cot(k_2R + \delta) = k_1\cot k_1R$$

Again, we have a transcendental equation to solve; given E (which we control through the energy of the incident particle), V_0, and R, we can in principle solve for δ

$$\psi_{\text{incident}} = Ae^{ikz}$$

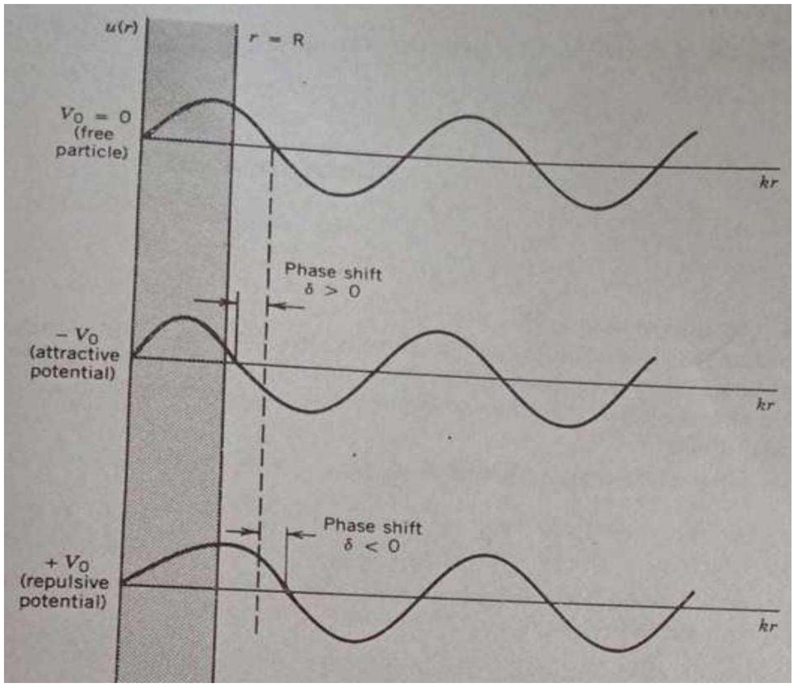
Let the target be located at the origin. Multiplying by the time-dependent factor

$$\psi(z,t) = Ae^{i(kz-\omega\ )}$$

which always moves in the $+z$ direction (toward the target for $z < 0$ and away

$$\psi_{\text{incident}} = \frac{A}{2ik}\left[\frac{e^{ikr}}{r} - \frac{e^{-ikr}}{r}\right]$$

The minus sign between the two terms keeps ψ finite for $r \to 0$, and using the coefficient A for both terms sets the amplitudes of the incoming and outgoing waves to be equal.



The effect of a scattering potential is to shift the phase of the scattered wave at points beyond the scattering regions, where the wave function is that of a free particle.

$$\psi(r)=\frac{A}{2ik}\left[\frac{e^{i(kr+\beta)}}{r}-\frac{e^{-ikr}}{r}\right]$$

where β is the change in phase.

$$\begin{aligned}\psi(r)&=\frac{C}{r}\sin(kr+\delta_0)\\&=\frac{C}{r}\frac{e^{i(kr+\delta_0)}-e^{-i(kr+\delta_0)}}{2i}\end{aligned}$$

Thus $\beta=2\delta_0$ and $A=kCe^{-i\delta_0}$, for scattering, we need the amplitude to evaluate the probability for ψ represents all waves in the region $r>R$, scattered wave. The wave function the scattered wave we must subtract away the and to find the amp

$$\psi_{\text{scatitered}}=\psi-\psi_{\text{incident}}$$

The current of scattered particles per unit area can be found as extended to three dimensions:

$$\begin{aligned}j_{\text{seatered}}&=\frac{h}{2mi}\left(\psi^*\frac{\partial\psi}{\partial r}-\frac{\partial\psi^*}{\partial r}\psi\right)\\&=\frac{h|A|^2}{mkr^2}\sin^2\delta_0\end{aligned}$$

and the incident current is, in analogy with Equation

$$j_{\text{incident}} = \frac{\hbar k|A|^2}{m}$$

The scattered current is uniformly distributed over a sphere of radius r. An element of area $r^2 d\Omega$ on that sphere subtends a solid angle $d\Omega = \sin\theta d\theta d\phi$ at the scattering center.

$$d\sigma = \frac{(j_{\text{scateredd}})(r^2 d\Omega)}{j_{\text{incident}}}$$

Using Equations of the scattered and incident currents, we obtain

$$\frac{d\sigma}{d\Omega} = \frac{\sin^2\delta_0}{k^2}$$

The total cross section σ is the total probability to be scattered in any direction:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

Effective range theory

The low energy $n-p$ scattering cross-section

$$\sigma = \frac{4\pi}{k^2 + 1/a_k^2}$$

where a_k is the seattering length for the energy E and $k^2 = (ME/\hbar^2)$. The energy E_1 and E_2 for the wave equations

Consider two energies E_1 and E_2 for which the wave equations are

Consider two energin
$$u_1'' + \{k_1^2 - U(r)\}u_1 = 0$$
$$u_2'' + \{k_2^2 - U(r)\}u_2 = 0$$
where $k_1^2 = ME_1/\hbar^2; k_2^2 = ME_2/\hbar^2; U(r) = MV(r)/\hbar^2.$

Multiplying the first of the Eqs. (17.9-1) by u_2'' and the second by u_1'', we get

the first on $u_2 u_1'' + k_1^2 u_1 u_2 = U(r) u_1 u_2$
$$u_1 u_2'' + k_2^2 u_1 u_2 = U(r) u_1 u_2$$

Subtracting, we get

$$u_2 u_1'' + u_1 u_2'' = \frac{d}{dr}(u_2 u_1' - u_1 u_2') = (k_2^2 - k_1^2) u_1 u_2$$

Integrating we get

$$\int_0^r \frac{d}{dr}(u_2 u_1' - u_1 u_2') dr = (k_2^2 - k_1^2) \int_0^r u_1 u_2 dr$$

Since $u_1(0) = u_2(0) = 0$, we get

$$(u_2 u_1' - u_1 u_2')_r = (k_2^2 - k_1^2) \int_0^r u_1 u_2 dt$$

Let us now consider the asymptotic solutions of the wave eqn. Since U large r, the solutions are of the form:

$$v_1' = \frac{k_1 \cos(k_1 r + \delta_1)}{\sin \delta_1}, v_2' = \frac{k_2 \cos(k_2 r + \delta_2)}{\sin \delta_2}$$
$$v_1'(0) = k_1 \cot \delta_1, v_2'(0) = k_2 \cot \delta_2$$

$$(r_2'' - v_1' v_2')_r - (k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^r v_1 v_2 dr$$

If no^{w} we push up the upper limits of integration in above eqn to $r = \propto$,

$$(u_2 u_1' - u_1 u_2')_\infty = (k_2^2 - k_1^2) \int_0^\infty u_1 u_2 dr$$
$$(i_1 v_1' - v_1 v_2')_\infty - (k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^\infty v_1 v_2 dr$$

At large $r, u_1(r)$ and $u_2(r)$ reduce to the asymptotic forms $v_1(r)$ and $v_2(r)$ respectively.

$$-(k_1 \cot \delta_1 - k_2 \cot \delta_2) = (k_2^2 - k_1^2) \int_0^\infty (v_1 v_2 - u_1 u_2) dr$$

Let us now choose the energy $E_1 = 0$ so that $k_1 = 0$. Also let $E_2 = E$ and $k_2 = k$. Then

$$\lim_{k_1 \to 0} (k_1 \cot \delta_1) = -\frac{1}{a}$$

where a is the Fermi scattering length. Further

$$k_2 \cot \delta_2 = k \cot \delta = -\frac{1}{a_k}$$

We then get

$$\frac{1}{a} - \frac{1}{a_k} = k^2 \int_0^\infty (v_0 v - u_0 u) dr$$

Now both $u(r)$ and $v(r)$ depend on E and hence on k. Let us write

$$\int_0^\infty (v_0 v - u_0 u) dr = \frac{1}{2} \rho(0, E)$$

where $\rho(0, E)$ depends on the energies $E_1 (= 0)$ and $E_2 (= E)$. So we get

$$\frac{1}{a_k} = \frac{1}{a} - \frac{1}{2} \rho(0, E) k^2$$

Since for small $r, V(r) > E$, the wave functions u_0 and v_0 for zero energy will differ very little from u and v respectively for the energy E so that for small r we can write $u(r) \approx u_0(r)$ and $v(r) \approx$

On the other hand, for large r (beyond the range of the $n - p$ interaction potential), $u(r)$ and $u_0(\mathrm{r}) \rightarrow v_0(r)$. So for some arbitrarily chosen range a of the potential we have

$$\int_0^a (v_0 v - u_0 u) dr = \int_0^a (v_0^2 - u_0^2) dr$$
$$\int_a^\infty (v_0 v - u_0 u) dr = 0$$

and

$$\int_a^\infty (v_0 v - u_0 u) dr = \int_0^\infty (v_0^2 - u_0^2) dr = \frac{1}{2} \rho(0,0) = \frac{1}{2} r_0$$

where r_0 is a constant known as the effective range of the interaction which gives an average rand and the proton. We get of the interaction between the neutron and the proton. Thus from Eq. (17.9-16) we get

$$\frac{1}{a_k} = \frac{1}{a} - \frac{1}{2} r_0 k^2$$

Spin dependence of nuclear force

This observation is from the failure to observe a single bound state of the deuteron and also measured from the difference between the singlet and triplet cross section. The term must only depend on the two nucleons s_1 and s_2 but not all the combinations of s_1 and s_2 are not possible.

Nuclear force must satisfy certain symmetries, which restrict the possible form that a potential could have. Examples of these symmetries are purity (r→ -r) and time reversal (t→-t). Experiments say that to high degree of precision, internuclear potential is invariant with respect to those operations. Upon time reversal, all motions are reversed. Thus, terms such as s_1 or s_2 or linear combination As_1+Bs_2 in potential would violate time reversal invariance and cannot be part of nuclear potential terms such as s_1^2, s_2^2 or $s_1.s_2$ are invariant with respect to time reversal and are allowed. The simplest term involving both nucleons is $s_1.s_2$

Let us consider the value of $s_1.s_2$ for singlet and triplet states. We evaluate the total spins $S=s_1+s_2$

$$S^2 = S * S = (s_1 + s_2).(s_1 + s_2)$$

Thus,$s_1.s_2 = \frac{1}{2}(S^2 - s_1^2 - s_2^2)$

The squared angular momenta $s^2 = \hbar s(s+1)$

$$< s_1.s_2 >= \frac{1}{2}[S(S+1) - s_1(s_1+1) - s_2(s_2+1)]\hbar$$

Nuclear spin $s_1$and s_2 is $\frac{1}{2}$

For S=1

$$< s_1.s_2 \geq \frac{1}{2}\left[1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]\hbar = \frac{1}{4}\hbar$$

For S=0

$$< s_1.s_2 \geq \frac{1}{2}\left[0(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]\hbar = -\frac{3}{4}\hbar$$

Spin-dependent expression $s_1.s_2V_s(r)$ can be included in potential. The magnitude of V_s can be adjusted to give correct difference between singlet and triplet cross section. Radial dependence can be adjusted to give the proper dependence of energy.

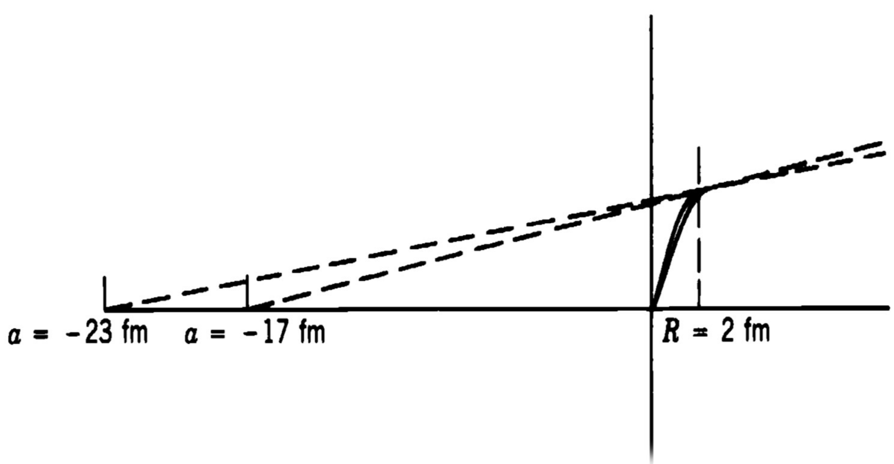
$$V_{(r)=} - \left(\frac{s_1.s_2}{\hbar^2} - \frac{1}{4}\right)V_1(r) + \left(\frac{s_1.s_2}{\hbar^2} - \frac{3}{4}\right)V_3(r)$$

$V_1(r)$ and $V_2(r)$ are potentials that separately give proper singlet and triplet behaviour.

Charge independence and charge symmetry

The three nuclear forces nn, pp, and pn are identical, again correcting for the pp Coulomb force. Charge independence is thus a stronger requirement than charge symmetry. The singlet np scattering length (- 23.7 fm) seems to differ substantially from the pp and nn scattering lengths (- 17 fm).

From Figure, large negative scattering lengths are extraordinarily sensitive to the nuclear wave function near ***r*** = ***R*,** and a very small change in ψ can give a large change in the scattering length. Thus, the large difference between the scattering lengths may correspond to a very small difference between the potentials, which is easily explained by the exchange force model.



The proton-proton interaction is identical to the neutron-neutron interaction, after we correct for the Coulomb force in the proton-proton system. Here "charge" refers to the character of the nucleon and not to electric charge.

The pp parameters must first be corrected for the Coulomb interaction. When this is done, the, resulting singlet pp parameters are

$$a = -17.1 \pm 0.2 \text{ fm}$$

$$r_0 = 2.84 \pm 0.03 \text{ fm}$$

These are in very good agreement with the measured nn parameters

$$a = -16.6 \pm 0.5 \text{ fm}$$

$$r_0 = 2.66 \pm 0.15 \text{ fm}$$

which strongly supports the notion of charge symmetry.

Isospin formalism

The concept of isospin in nuclear physics was introduced by Heisenberg (1932). Recently, with the acquisition of large bodies of nuclear data, the relevance of isospin has been firmly established and its significance in the investigation of the atomic nucleus has been clearly brought out. The topic is now in the process of active development, due to the vigorous efforts of research laboratories all over the world. This paper will briefly discuss the role of isospin in understanding the structure of the nucleus and the contributions currently being made in this field by the Nuclear Physics Research Unit at the University of the Witwatersrand. A short review of the fundamentals of isospin will also be given.

Heisenberg's theory recognized that the neutron (n) and the proton (p) are so similar that they can be represented as two quantum states of one particle called the nucleon. The major difference between neutrons and protons is of course electric charge; this is however a small perturbation compared with the nuclear force. The theory was taken further by Wigner (1937), using the mathematical analogy of ordinary spin or angular momentum of a particle. The quantum number 'isospin' (T) was introduced to describe the two different charge states of the nucleon. The quantum number T has the following component along the z-axis in isospin-space:

$$\begin{aligned} T_Z &= +1/2 \text{ for the neutron (n)} \\ &= -1/2 \text{ for the proton (p).} \end{aligned}$$

If one sums the z-components of isospin for all the constituent nucleons making up a nucleus of mass number A, one obtains the total T_Z for that nucleus,

i.e., for a nucleus,
$$T_Z = \sum_{i}^{A} T_Z^i = \frac{N - Z}{2}$$

where N is the number of neutrons and Z the number of protons (or atomic number) in the nucleus, T_Z^i being the individual isospin values of the nucleons.

UNIT III: NUCLEAR REACTIONS

Kinds of nuclear reactions – Reaction kinematics – Q-value – Partial wave analysis of scattering and reaction cross section – scattering length– Compound nuclear reactions – Reciprocity theorem – Resonances –Breit Wigner one level formula – Direct reactions - Nuclear Chain reaction – four factor formula.

Kinds of nuclear reactions

A typical nuclear reaction is written

$$a + X \rightarrow Y + b$$

where a is the accelerated projectile, X is the target (usually stationary in the laboratory), and Y and b are the reaction products. Usually, Y will be a heavy product that stops in the target and is not directly observed, while b is a light particle that can be detected and measured. Generally, a and b will be nucleons or light nuclei, but occasionally b will be a γ ray, in which case the reaction is called radiative capture. (If a is a γ ray, the reaction is called the nuclear photoeffect.)

An alternative and compact way of indicating the same reaction is

$$X(a, b)Y$$

which is convenient because it gives us a natural way to refer to a general class of reactions with common properties, for example (α, n) or (n, γ) reactions.

We classify reactions in many ways. If the incident and outgoing particles are the same (and correspondingly X and Y are the same nucleus), it is a scattering process, elastic if Y and b are in their ground states and inelastic if Y or b is in an excited state (from which it will generally decay quickly by γ emission). Sometimes a and bare the same particle, but the reaction causes yet another nucleon to be ejected separately (so that there are three particles in the final state); this is called a knockout reaction. In a transfer reaction, one or two nucleons are transferred between projectile and target, such as an incoming deuteron turning into an outgoing proton or neutron, thereby adding one nucleon to the target X to form Y . Reactions can also be classified by the mechanism that governs the process. In direct reactions (of which transfer reactions are an important subgroup), only very few nucleons take part in the reaction, with the remaining nucleons of the target serving as passive spectators. Such reactions might insert or remove a single nucleon from a shell-model state and might therefore serve as ways to explore the shell structure of nuclei. Many excited states of Y can be reached in these reactions. The other extreme is the compound nucleus mechanism, in which the incoming and target nuclei merge briefly for a complete sharing of energy before the outgoing nucleon is ejected, somewhat like evaporation of a molecule from a hot liquid. Between these two extremes are the resonance reactions, in which the incoming particle forms a "quasi bound" state before the outgoing particle is ejected.

Observables

We have at our disposal techniques for measuring the energies of the outgoing particles to high precision (perhaps 10 keV resolution with a magnetic spectrometer). We can determine the direction of emission of the outgoing particle, and observe its angular distribution (usually relative to the axis of the original beam) by counting the number emitted at various angles. The differential cross section is obtained from the probability to observe particle b with a certain energy and at a certain angle (θ, ϕ) with respect to the beam axis. Integrating the differential cross section over all angles, we get the total cross section for particle b to be emitted at a certain energy (which is also sometimes called a differential cross section). We can also integrate over all energies of b to get the absolute total cross section, which is in effect the probability to form nucleus Y in the reaction. This quantity is of interest in, for instance, neutron activation or radioisotope production.

By doing polarization experiments, we can deduce the spin orientation of the product nucleus Y or perhaps the spin dependence of the reaction cross section. For these experiments we may need an incident beam of polarized particles, a target of polarized nuclei, and a spectrometer for analyzing the polarization of the outgoing particle b.

We can simultaneously observe the γ radiations or conversion electrons from the decay of excited states of Y. This is usually done in coincidence with the particle b to help us decide which excited states the radiations come from. We can also observe the angular distribution of the γ radiations, as an aid in interpreting the properties of the excited states, especially in deducing their spin-parity assignments.

Reaction kinematics and q value

Conservation of total relativistic energy in our basic reaction gives

$$m_{\mathrm{x}} c^2+T_{\mathrm{X}}+m_{\mathrm{a}} c^2+T_{\mathrm{a}}=m_{\mathrm{Y}} c^2+T_{\mathrm{Y}}+m_{\mathrm{b}} c^2+T_{\mathrm{b}}$$

where the T 's are kinetic energies (for which we can use the nonrelativistic approximation $\frac{1}{2} m v^2$ at low energy) and the m 's are rest masses. We define the reaction Q value, in analogy with radioactive decay Q values, as the initial mass energy minus the final mass energy:

$$Q=\left(m_{\text {initial }}-m_{\text {final }}\right) c^2$$

which is the same as the excess kinetic energy of the final products:

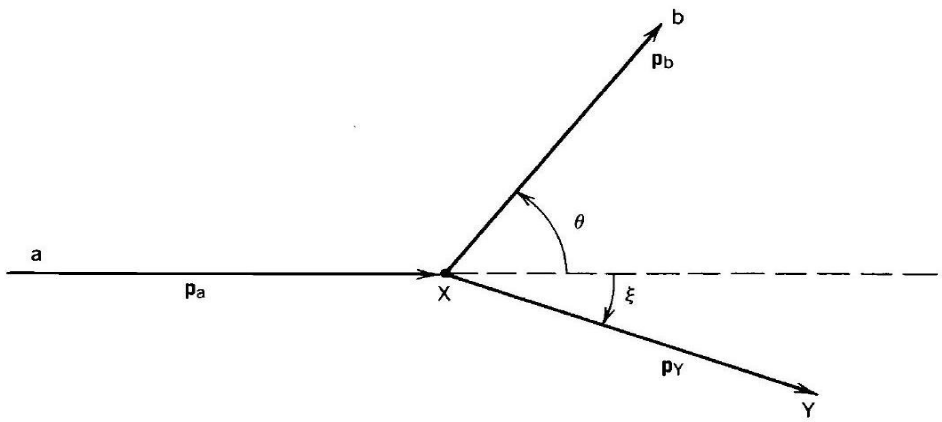
$$Q=T_{\text {final }}-T_{\text {initial }}$$

The Q value may be positive, negative, or zero. If $Q>0\left(m_{\text {initial }}>m_{\text {final }}\right.$ or $T_{\text {final }}>$ $T_{\text {initial }}$) the reaction is said to be exoergic or exothermic; in this case nuclear mass or binding energy is released as kinetic energy of the final products. When $Q<0\left(m_{\text {initial }}<m_{\text {final }}\right.$ or $\left.T_{\text {final }}<T_{\text {initial }}\right)$ the reaction is endoergic or endothermic, and initial kinetic energy is converted into nuclear mass or binding energy. The changes in mass and energy must of course be related by the familiar expression from special relativity, $\Delta E=\Delta m c^2$-any change in the kinetic energy of the system of reacting particles must be balanced by an equal change in its rest energy.

These are valid in any frame of reference in which we choose to work. Let's apply them first to the laboratory reference frame, in which the target nuclei are considered to be at rest (room-temperature thermal energies are negligible on the MeV scale of nuclear reactions). If we define a reaction plane by the direction of the incident beam and one of the outgoing particles, then conserving the component of momentum perpendicular to that plane shows immediately that the motion of the second outgoing particle must lie in the plane as well. Figure shows the basic geometry in the reaction plane. Conserving linear momentum along and perpendicular to the beam direction gives

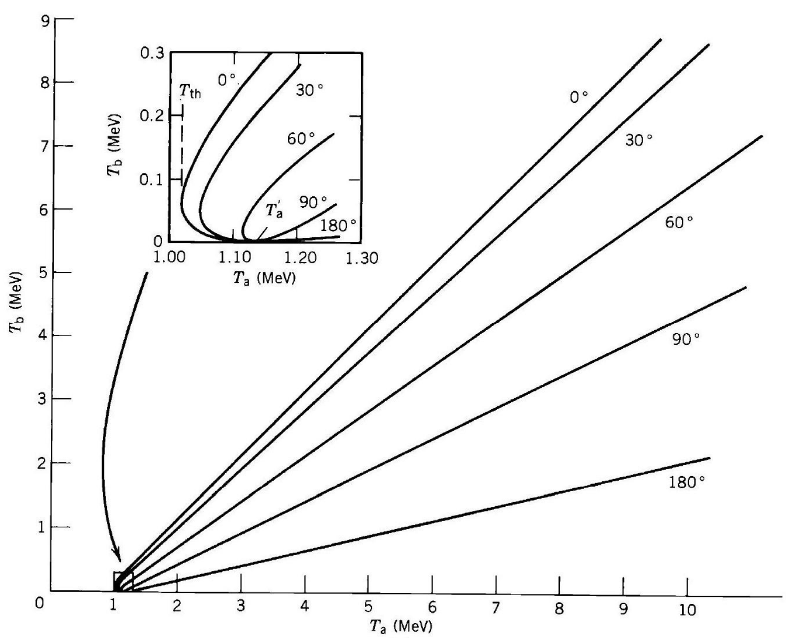
$$\begin{aligned} p_{\mathrm{a}} & =p_{\mathrm{b}} \cos \theta+p_{\mathrm{Y}} \cos \xi \\ 0 & =p_{\mathrm{b}} \sin \theta-p_{\mathrm{Y}} \sin \xi \end{aligned}$$

Regarding Q as a known quantity and T_a (and therefore p_a) as a parameter that we control, these equations represent three equations in four unknowns (θ, ξ, T_b, and T_Y) which have no unique solution. If, as is usually the case, we do not observe Y, we can eliminate ξ and T_Y from



equation.

Basic reaction geometry for a + X → b + Y.



T_a vs T_b for the reaction $^3H(p, n)^3He$. The inset shows the region of double-valued behavior near 1.0 MeV.

to find a relationship between T_{b} and θ :

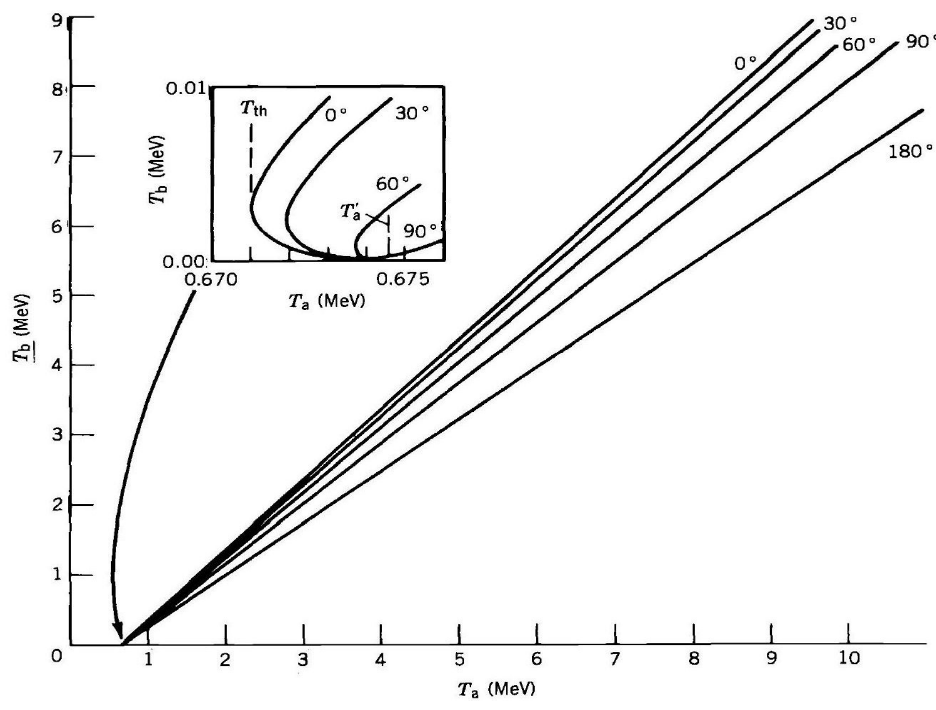
$$T_{\mathrm{b}}^{1/2}=\frac{(m_{\mathrm{a}}m_{\mathrm{b}}T_{\mathrm{a}})^{1/2}\cos\theta\pm\{m_{\mathrm{a}}m_{\mathrm{b}}T_{\mathrm{a}}\cos^2\theta+(m_{\mathrm{Y}}+m_{\mathrm{b}})[m_{\mathrm{Y}}Q+(m_{\mathrm{Y}}-m_{\mathrm{a}})T_{\mathrm{a}}]\}^{1/2}}{m_{\mathrm{Y}}+m_{\mathrm{b}}}$$

This expression is plotted in Figure $11.2a$ for the reaction $^{3}\mathrm{H(p,n)}\,^{3}\mathrm{He}$, for which $Q=-763.75\mathrm{keV}$. Except for a very small energy region between 1.019 and 1.147 MeV, there is a one-to-one correspondence (for a given T_{a}) between T_{b} and θ. That is, keeping the incident energy fixed, choosing a value of θ to observe the outgoing particles automatically then selects their energy.

1. There is an absolute minimum value of T_{a} below which the reaction is not possible. This occurs only for $Q<0$ and is called the threshold energy T_{th} :

$$T_{\mathrm{th}}=(-Q)\frac{m_{\mathrm{Y}}+m_{\mathrm{b}}}{m_{\mathrm{Y}}+m_{\mathrm{b}}-m_{\mathrm{a}}}$$

The threshold condition always occurs for $\theta=0^{\circ}$ (and therefore $\xi=0^{\circ}$)—the products Y and b move in a common direction (but still as separate nuclei).



T_a vs T_b for the reaction $^{14}\mathrm{C(p,n)}\,^{14}\mathrm{N}$. The inset shows the double-valued region.

No energy is "wasted" in giving them momentum transverse to the beam direction. If $Q>0$, there is no threshold condition and the reaction will "go" even for very small energies, although we may have to contend with Coulomb barriers not considered here and which will tend to keep a and X outside the range of each other's nuclear force.

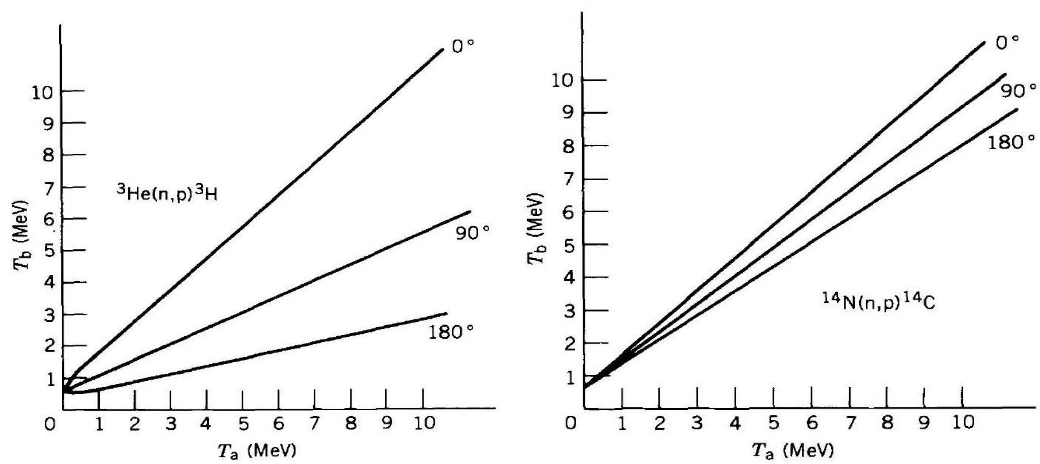
2. The double-valued situation occurs for incident energies between T_{th} and the upper limit

$$T_{\mathrm{a}}' = (-Q)\frac{m_{\mathrm{Y}}}{m_{\mathrm{Y}} - m_{\mathrm{a}}}$$

This also occurs only for $Q < 0$, and is important only for reactions involving nuclei of comparable masses. Using Equations 11.6 and 11.7 we can approximate this range as

$$T_{\mathrm{a}}' - T_{\mathrm{th}} \cong T_{\mathrm{th}}\frac{m_{\mathrm{a}}m_{\mathrm{b}}}{m_{\mathrm{Y}}(m_{\mathrm{Y}} - m_{\mathrm{a}})}\left(1 - \frac{m_{\mathrm{b}}}{m_{\mathrm{Y}}} + \cdots\right)$$

and you can see that if a and b have mass numbers of 4 or less and if Y is a medium or heavy nucleus, then the range ($T_{\mathrm{a}}' - T_{\mathrm{th}}$) becomes much smaller



T_{a} vs T_{b} for the reactions $^{3}\mathrm{He(n,p)}\,^{3}\mathrm{H}$ and $^{14}\mathrm{N(n,p)}^{14}\mathrm{C}$. No double-valued behavior occurs.

than 1% of the threshold energy. Figure $11.2b$ shows the double-valued region for the reaction $^{14}\mathrm{C(p,n)}\,^{14}\mathrm{N}$.

3. There is also a maximum angle θ_{m} at which this double-valued behavior occurs, the value for which is determined for T_{a} in the permitted range by the vanishing of the argument in the square root of Equation 11.5:

$$\cos^2\theta_m = -\frac{(m_Y + m_b)[m_Y Q + (m_Y - m_a)T_a]}{m_a m_b T_a}$$

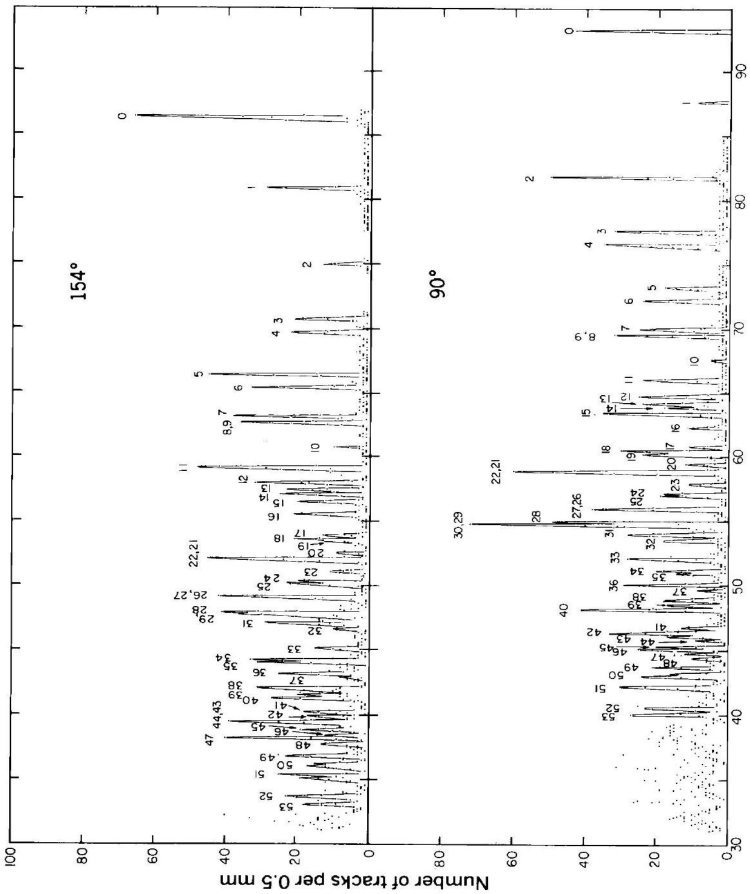
When $T_a = T'_a$, the double-valued behavior occurs between $\theta = 0°$ and $\theta_m = 90°$; near $T_a = T_{th}$ it occurs only near $\theta_m = 0°$.

4. Reactions with $Q > 0$ have neither a threshold nor a double-valued behavior, as you can see by reversing the reactions shown in Figures, $^3He(n,p)\,^3H$ and $^{14}N(n,p)\,^{14}C$, for which we can in each case make the single transformation $-Q \rightarrow +Q$. Figure 11.3 shows the T_b vs T_a graphs for these cases. The reactions occur down to $T_a \rightarrow 0$ (no threshold), and the curves are single-valued for all θ and T_a.

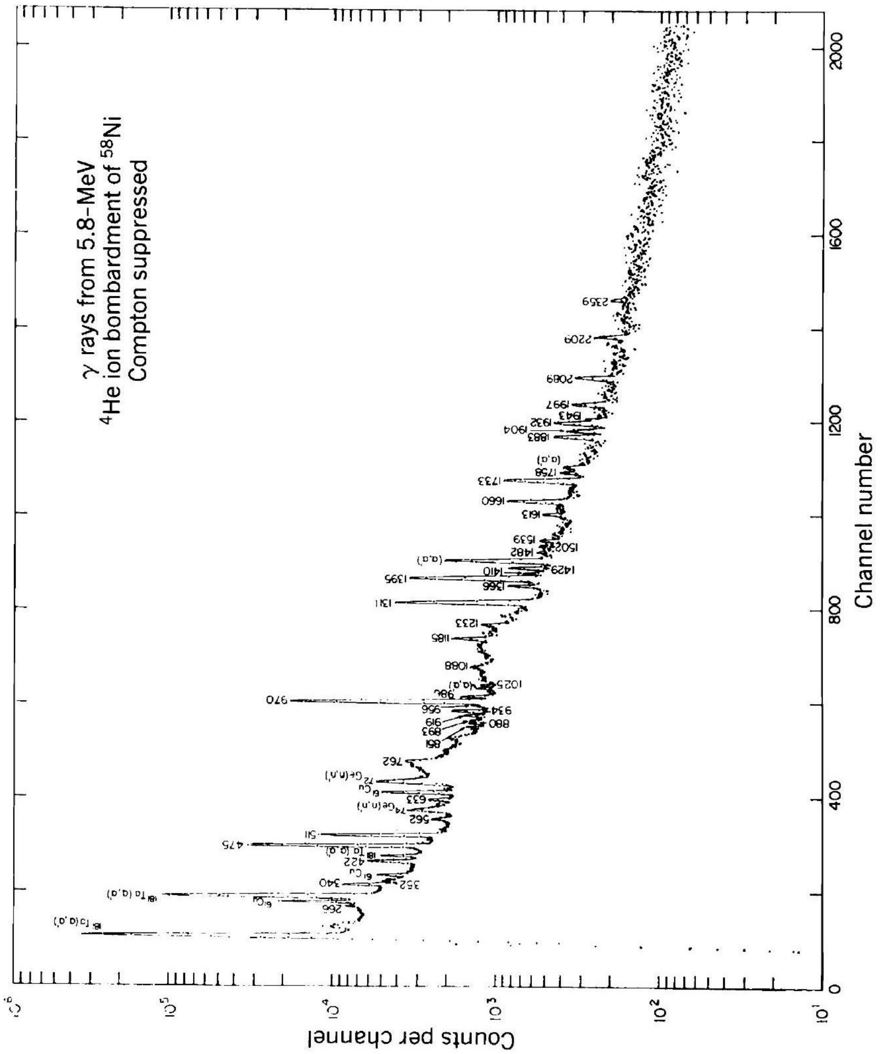
If, for a given θ and T_a, we measure T_b, then we can determine the Q value of the reaction and deduce the mass relationships among the constituents. If we know m_a, m_b, and m_X, we then have a way of determining the mass of Y. Solving Equation for Q, we obtain

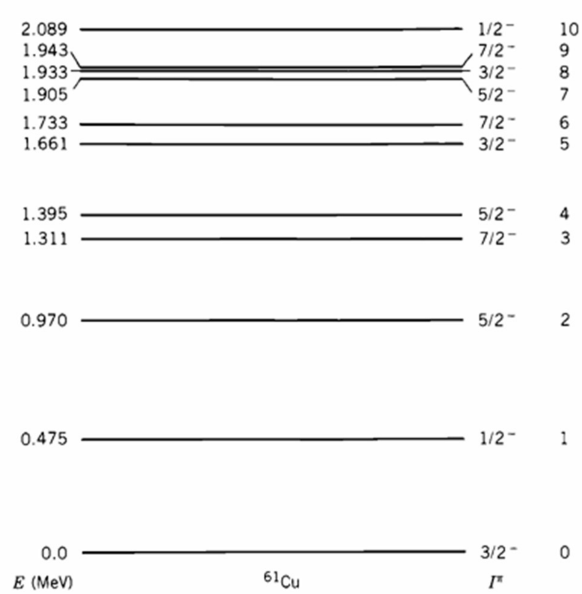
$$Q = T_b\left(1 + \frac{m_b}{m_Y}\right) - T_a\left(1 - \frac{m_a}{m_Y}\right) - 2\left(\frac{m_a}{m_Y}\frac{m_b}{m_Y}T_a T_b\right)^{1/2}\cos\theta$$

This procedure is not strictly valid, for m_Y also appears on the right side of the equation, but it is usually of sufficient accuracy to replace the masses with the integer mass numbers, especially if we measure at 90° where the last term vanishes.



Spectrum of protons from the reaction $^{58}Ni(^{4}He, p)\,^{61}Cu$. The highest energy proton group populates the ground state of ^{61}Cu, while the remaining groups lead to excited states (numbered 1,2,3, ..). The spectra taken at angles of 90 and $154°$ show a very dramatic angular dependence; note especially the change in the cross section for groups 1 and 2 at the two angles. (b) γ rays following the reaction. (c) Deduced partial level scheme of ^{61}Cu. Data from E. J. Hoffman, D. G. Sarantites, and N.-H. Lu, Nucl. Phys. A 173, 146 (1971).


γ rays from 5.8-MeV
^{4}He ion bombardment of ^{58}Ni
Compton suppressed
Channel number
Counts per channel


2.089 1/2⁻ 10
1.943 7/2⁻ 9
1.933 3/2⁻ 8
1.905 5/2⁻ 7
1.733 7/2⁻ 6
1.661 3/2⁻ 5
1.395 5/2⁻ 4
1.311 7/2⁻ 3
0.970 5/2⁻ 2
0.475 1/2⁻ 1
0.0 3/2⁻ 0
E (MeV)
^{61}Cu
I^π

As an example of the application of this technique, we consider the reaction $^{26}\mathrm{Mg}(^{7}\mathrm{Li},{}^{8}\mathrm{B})^{25}\mathrm{Ne}$. The nucleus $^{26}\mathrm{Mg}$ already has a neutron excess, and the removal of two additional protons in the reaction results in the final nucleus $^{25}\mathrm{Ne}$ with a large excess of neutrons. Data reported by Wilcox et al., Phys. Rev. Lett. 30, 866 (1973), show a $^{8}\mathrm{B}$ peak about 55.8 MeV observed at a lab angle of 10° when the incident $^{7}\mathrm{Li}$ beam energy is 78.9 MeV . Using Equation with mass numbers instead of masses gives $Q=-22.27\mathrm{MeV}$, which gives 24.99790 u for the mass of $^{25}\mathrm{Ne}$. Iterating the calculation a second time with the actual masses instead of the mass numbers does not change the result even at this level of precision.

If the reaction reaches excited states of Y , the Q-value equation should include the mass energy of the excited state.

$$Q_{\mathrm{ex}}=(m_{\mathrm{X}}+m_{\mathrm{a}}-m_{\mathrm{Y}}^{*}-m_{\mathrm{b}})c^{2}$$

where Q_0 is the Q value corresponding to the ground state of Y , and where we have used $m_{\mathrm{Y}}^{*}c^{2}=m_{\mathrm{Y}}c^{2}+E_{\mathrm{ex}}$ as the mass energy of the excited state (E_{ex} is the excitation energy above the ground state). The largest observed value of T_{b} is normally for reactions leading to the ground state, and we can thus use Equation to find Q_0. Successively smaller values of T_{b} correspond to higher excited states, and by measuring T_{b} we can deduce Q_{ex} and the excitation energy E_{ex}.

Figure shows an example of this kind of measurement. The peaks in the figure serve to determine T_{b}, from which the following Q values and excited-state.

Partial wave analysis of scattering and reaction cross sections

In this section we cover some details of reaction cross sections more thoroughly. We take the z axis to be the direction of the incident beam and assume it can be represented by a plane wave e^{ikz} corresponding to momentum $p=\hbar k$. The outgoing particles will be represented by spherical waves, and so the manipulations become easier if we express the incident plane wave as a superposition of spherical waves:

$$\psi_{\mathrm{inc}}=Ae^{ikz}=A\sum_{\ell=0}^{\infty} i^{\ell}(2\ell+1)j_{\ell}(kr)P_{\ell}(\cos\theta)$$

where A is an appropriately chosen normalization constant. The radial functions $j_\ell(kr)$ are spherical Bessel functions which were previously given in Table 2.3; they are solutions to the radial part of the Schrödinger equation, Equation 2.60, in a region far from the target where the nuclear potential vanishes. The angular functions $P_\ell(\cos\theta)$ are Legendre polynomials:

$$P_0(\cos\theta) = 1$$
$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

This expansion of the incident (and eventually, the scattered) wave is called the partial wave expansion, with each partial wave corresponding to a specific angular momentum ℓ. Such a procedure is valid if the nuclear potential is assumed to be central. What makes the method useful is that it is often sufficient to consider the effect of the nuclear potential on at most only a few of the lowest partial waves (such as $\ell = 0$ or s-wave nucleon-nucleon scattering discussed in Chapter 4). If the particles of momentum p interact with impact parameter b, then the (semiclassical) relative angular momentum will be

$$\ell\hbar = pb$$

or

$$b = \ell\frac{\hbar}{p} = \ell\frac{\lambda}{2\pi} = \ell\lambda$$

where $\lambda = \lambda/2\pi$ is called the reduced de Broglie wavelength. Incidently, $\lambda = k^{-1}$. According to quantum mechanics, ℓ can only be defined in integer units, and thus the semiclassical estimate should be revised somewhat. That is, particles with (semiclassical) angular momenta between $0\hbar$ and $1\hbar$ will interact through impact parameters between 0 and λ, and thus effectively over an area (cross section) of at most $\pi\lambda^2$. With $\hbar \le \ell \le 2\hbar$, the cross section is a ring of inner radius λ and outer radius 2λ, and thus of area $3\pi\lambda^2$. We can thus divide the interaction area into a number of zones, each corresponding to a specific angular momentum ℓ and each having area $\pi[(\ell+1)\lambda]^2 - \pi(\ell\lambda)^2 = (2\ell+1)\pi\lambda^2$. We can estimate the maximum impact parameter for nuclear scattering to be about $R = R_1 + R_2$ (the sum of the radii of the incident and target nuclei), and thus the maximum ℓ value likely to occur is R/λ, and the total cross section is correspondingly

$$\sigma = \sum_{\ell=0}^{R/\lambda} (2\ell + 1)\pi\lambda^2 = \pi(R + \lambda)^2$$

This is a reasonable estimate, for it includes not only an interaction distance R, but it allows the incident particle's wave nature to spread over a distance of the order of λ, making the effective interaction radius $(R + \lambda)$. We will see later how the exact calculation modifies this estimate.

When the wave is far from the nucleus, the $j_\ell(kr)$ have the following convenient expansion:

$$j_\ell(kr) \cong \frac{\sin\left(kr - \frac{1}{2}\ell\pi\right)}{kr} \quad (kr \gg \ell)$$

so that

$$\psi_{\text{inc}} = \frac{A}{2kr}\sum_{\ell=0}^{\infty} i^{\ell+1}(2\ell + 1)\left[e^{-i(kr-\ell\pi/2)} - e^{+i(kr-\ell\pi/2)}\right]P_\ell(\cos\theta)$$

The first term in brackets, involving e^{-ikr}, represents an incoming spherical wave converging on the target, while the second term, in e^{+ikr}, represents an outgoing spherical wave emerging from the target nucleus. The superposition of these two spherical waves, of course, gives the plane wave.

The scattering can affect only the outgoing wave, and can affect it in either of two ways: through a change in phase and through a change in amplitude. The change in amplitude suggests that there may be fewer particles coming out than there were going in, which may appear to be a loss in the net number of particles. However, keep in mind that the wave function represents only those particles of momentum $\hbar k$. If there is inelastic scattering (or some other nuclear reaction), the energy (or even the identity) of the outgoing particle may change. It is therefore not surprising that there may be fewer particles in the e^{ikr} term following inelastic scattering. It has become customary to refer to a specific set of conditions (exclusive of direction of travel) of the outgoing particle and residual nucleus as a reaction channel. The reaction may thus proceed through the elastic channel or through any one of many inelastic channels. Some channels may be closed to the reacting particles, if there is not enough energy or angular momentum to permit a specific final configuration to be reached.

We account for the changes in the ℓ th outgoing partial wave by introducing the complex coefficient η_ℓ into the outgoing (e^{ikr}) term

$$\psi = \frac{A}{2kr}\sum_{\ell=0}^{\infty} i^{\ell+1}(2\ell+1)\left[e^{-i(kr-\ell\pi/2)} - \eta_\ell e^{+i(kr-\ell\pi/2)}\right]P_\ell(\cos\theta)$$

This wave represents a superposition of the incident and scattered waves: $\psi = \psi_{\text{inc}} + \psi_{\text{sc}}$. To find the scattered wave itself, we subtract equations

$$\psi_{\text{sc}} = \frac{A}{2kr}\sum_{\ell=0}^{\infty} i^{\ell+1}(2\ell+1)(1-\eta_\ell)e^{i(kr\quad\ell\pi/2)}P_\ell(\cos\theta)$$

Because we have accounted for only those parts of ψ_{sc} with wave number k identical with the incident wave, this represents only elastic scattering. We now find the scattered current density:

$$j_{\text{sc}} = \frac{\hbar}{2mi}\left(\psi_{\text{sc}}^*\frac{\partial\psi_{\text{sc}}}{\partial r} - \frac{\partial\psi_{\text{sc}}^*}{\partial r}\psi_{\text{sc}}\right)$$

$$= |A|^2\frac{\hbar}{4mkr^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)i(1-\eta_\ell)P_\ell(\cos\theta)\right|^2$$

The incident current

$$j_{\text{inc}} = \frac{\hbar k}{m}|A|^2$$

the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)i(1-\eta_\ell)P_\ell(\cos\theta)\right|^2$$

To find the total cross section, we require the integral of the Legendre polynomials:

$$\int P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta d\theta d\phi = \frac{4\pi}{2\ell+1} \qquad \text{if } \ell = \ell'$$

Thus

$$\sigma_{\text{sc}} = \sum_{\ell=0}^{\infty} \pi \lambda^2 (2\ell + 1)|1 - \eta_\ell|^2$$

If elastic scattering were the only process that could occur, then $|\eta_\ell| = 1$ and it is conventional to write $\eta_\ell = e^{2i\delta_\ell}$ where δ_ℓ is the phase shift of the ℓ th partial wave. For this case, $|1 - \eta_\ell|^2 = 4\sin^2 \delta_\ell$ and

$$\sigma_{\text{sc}} = \sum_{\ell=0}^{\infty} 4\pi \lambda^2 (2\ell + 1)\sin^2 \delta_\ell$$

which reduces directly to Equation 4.30 for $\ell = 0$. If there are other processes in addition to elastic scattering (inelastic scattering or other reactions) then it is not valid, because $|\eta_\ell| < 1$. We group all of these processes together under the term reaction cross section σ_{r}, where we take "reaction" to mean all nuclear processes except elastic scattering. To find this cross section, we must examine above equations to find the rate at which particles are "disappearing" from the channel with wave number k. That is, we find the difference between the incoming current and the outgoing current.

$$|j_{\text{in}}| - |j_{\text{out}}| = \frac{|A|^2 \hbar}{4mkr^2}\{\left|\sum_{\ell=0}^{\infty} (2\ell + 1) i^{\ell+1} e^{i\ell\pi/2} P_\ell(\cos\theta)\right|^2$$

and the reaction cross section becomes

$$\sigma_{\text{r}} = \sum_{\ell=0}^{\infty} \pi \lambda^2 (2\ell + 1)(1 - |\eta_\ell|^2)$$

The total cross section, including all processes, is

$$\sigma_{\text{t}} = \sigma_{\text{sc}} + \sigma_{\text{r}}$$

You should note the following details about these results:

1. It is possible to have elastic scattering in the absence of other processes; that is, if $|\eta_\ell| = 1$, then it vanishes. It is not possible, however, to have reactions without also having elastic scattering; that is, any choice of η_ℓ for which $\sigma_{\text{r}} \neq 0$ for a given partial wave automatically gives $\sigma_{\text{sc}} \neq 0$ for that partial wave. We can understand this with reference to the diffraction model of scattering we considered. If particles are removed from the

incident beam, creating a "shadow" behind the target nucleus, incident particles will be diffracted into the shadow.

2. For a "black disk" absorber, in which all partial waves are completely absorbed up to $\ell = R/\lambda(\eta_\ell = 0$ for complete absorption) and unaffected for $\ell > R/\lambda(\eta_\ell = 1)$, then

$$\sigma_{\rm sc} = \pi(R + \lambda)^2$$

and

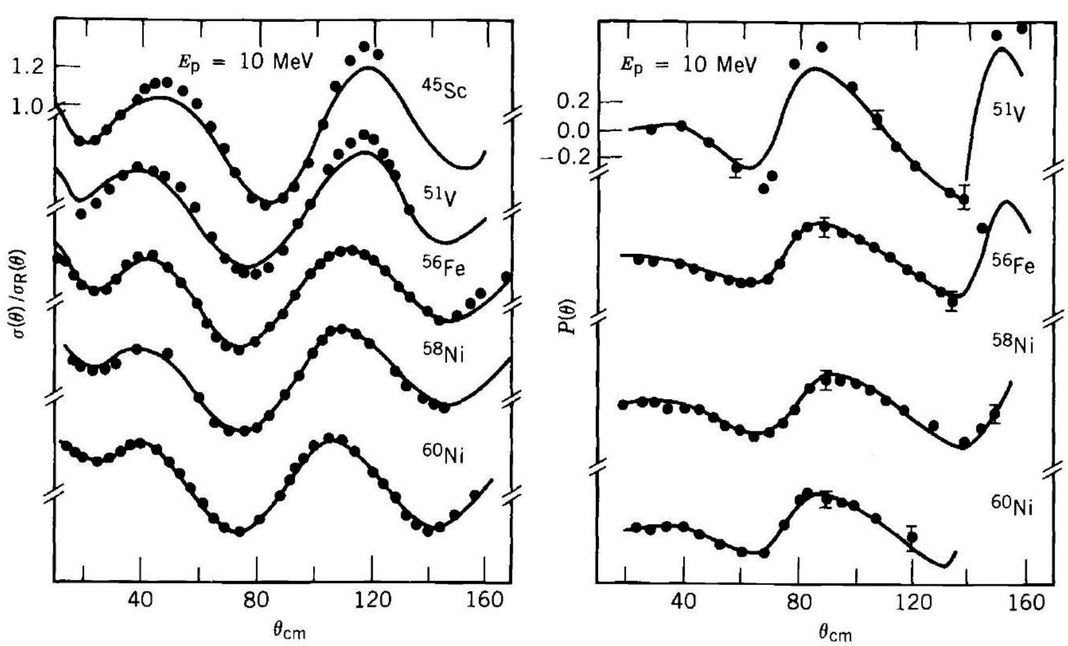
$$\sigma_{\rm r} = \pi(R + \lambda)^2$$

so that

$$\sigma_{\rm t} = 2\pi(R + \lambda)^2$$

The total cross section is twice the geometrical area. The explanation for this nonclassical effect can also be found in the "shadow" region-the target nucleus cannot simply absorb and throw a sharp shadow. It must also diffract into the shadow region.

The program for using these results to study nuclear structure is similar to that. For nucleon-nucleon scattering. We can guess at a form for the nuclear potential, solve the Schrödinger equation inside the interaction region



Optical-model fits to differential cross sections (at left, shown as a ratio to the Rutherford cross section) and polarizations, for $10-$ MeV protons scattered elastically from various targets. The solid lines are the fits to the data using the best set of optical-model parameters. From F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

Scattering length

Fermi and Marshall introduced a very useful concept the 'scattering length a ' for the discussion of nuclear scattering at very low incident neutron energy.

$$[\text{i.e. } E \rightarrow 0 \text{ and hence } k=\sqrt{\left\{\left(\frac{M E}{h^2}\right)\right\}} \rightarrow 0$$

Which may be defined as follows:

$$a=\operatorname{Lim}_{k \rightarrow 0}\left(-\frac{\sin \delta_0}{k}\right)$$

By this definition, equation which gives the total scattering cross section for S -wave ($\mathrm{l}=0$) can be written for very low incident neutron energy as

$$\operatorname{Lim}_{k \rightarrow 0}\left(\sigma_{s c}, o\right)=\operatorname{Lim}_{k \rightarrow 0}\left(\frac{4 \pi \sin ^2 \delta_0}{k^2}\right)=4 \pi a^2$$

Equation therefore shows that " a " has the dimensions of length, hence the name scattering length, and has the geometric significance of being the radius of a hard sphere around the scattering center from which neutrons are scattered.

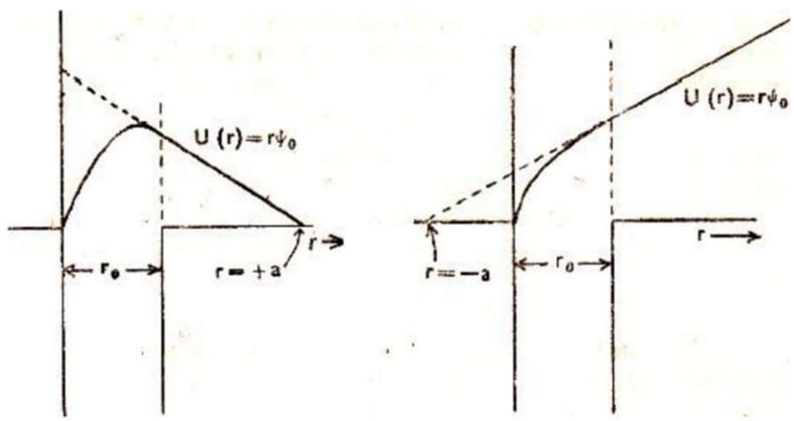
Now, it should be noticed from equation that as the energy E of the incident neutron approaches 0 , it must also approach either 0 or else the cross-section at zero neutron energy would become infinite, which is physically absurd. Therefore, at very low incident neutron energies $(E \rightarrow 0)$, equation reduces to

$$a=-\frac{\delta_0}{k}$$

Then at very low incident neutron energies, the wave function outside the range of nuclear force as expressed by equation may be written as

$$\mathrm{Lim}_{k\to 0}U(r) = \mathrm{Lim}_{k\to 0}(r\psi_0) = \mathrm{Lim}_{k\to 0}\left[e^{i\delta_0}\frac{\sin(kr+\delta_0}{k}\right]$$

The scattering length is then simply represented graphically by the equation. The scattering length ' a ' is the intercept on the r-axis and this equation depicts a straight line for U (r). Figure illustrates this. The relevance of positive or negative scattering length is that it informs us what is the significance of attaching a positive or negative sign with at the scattering length, an inquisitive reader may inquire quite naturally after we have defined the scattering length using equations. whether a bound or unbound state exists in the system.



Graphical interpretation of scattering length

It is evident from above fig that a positive scattering length denotes a bound state while a negative scattering length denotes an unbound or virtual state.

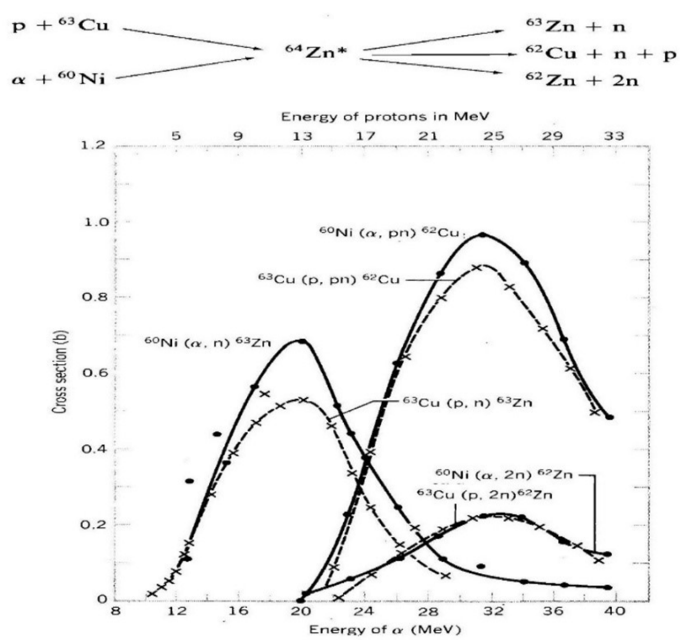
Compound-nucleur reactions

Suppose an incident particle enters a target nucleus with an impact parameter small compared with the nuclear radius. It then will have a high probability of interacting with one of the nucleons of the target, possibly through a simple scattering. The recoiling struck nucleon and the incident particle (now with less energy) can each make successive collisions with other nucleons, and after several such interactions, the incident energy is shared among many of the nucleons of the combined system of projectile + target.

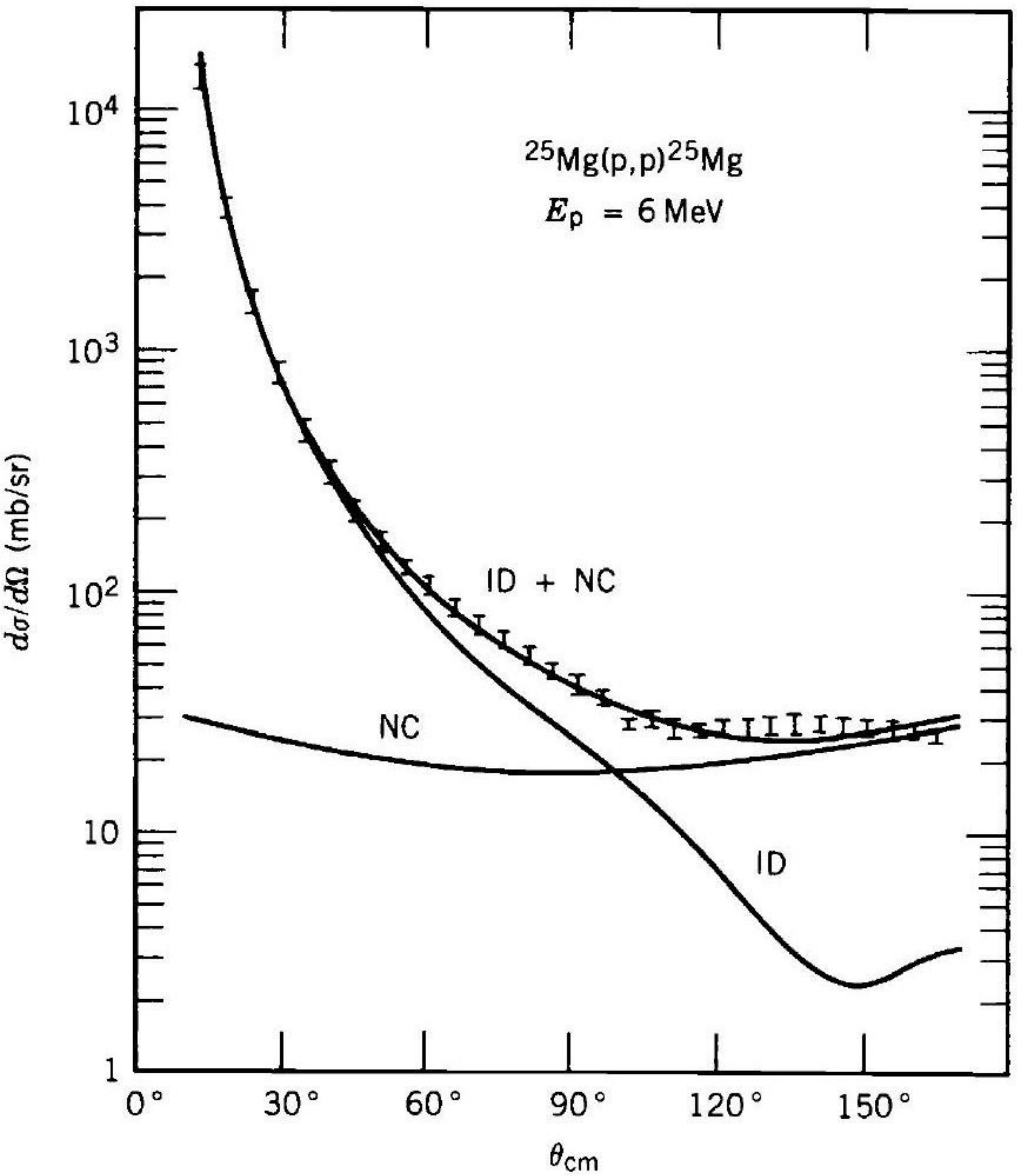
Such reactions have a definite intermediate state, after the absorption of the incident particle but before the emission of the outgoing particle (or particles). This intermediate state is called the compound nucleus. Symbolically then the reaction $\mathrm{a} + \mathrm{X} \rightarrow \mathrm{Y} + \mathrm{b}$ becomes

$$\mathrm{a} + \mathrm{X} \rightarrow \mathrm{C}^* \rightarrow \mathrm{Y} + \mathrm{b}$$

where C^* indicates the compound nucleus. As might be assumed from seeing the reaction written in this form, we can consider a reaction that proceeds through the compound nucleus to be a two-step process: the formation and then the subsequent decay of the compound nucleus. The decay probability depends only on the total energy given to the system; in effect, the compound nucleus "forgets" the process of formation and decays governed primarily by statistical rules. The compound nucleus $^{64}\mathrm{Zn}^*$ can be formed through several reaction processes, including $\mathrm{p} + {}^{63}\mathrm{Cu}$ and $\alpha + {}^{60}\mathrm{Ni}$. It can also decay in a variety of ways, including $^{63}\mathrm{Zn} + \mathrm{n}$, $^{62}\mathrm{Zn} + 2\mathrm{n}$, and $^{62}\mathrm{Cu} + \mathrm{p} + \mathrm{n}$. That is



Cross sections for different reactions leading to the compound nucleus $^{64}\mathrm{Zn}$ show very similar characteristics, consistent with the basic assumptions of the compound nucleus model. From S. N. Goshal, Phys. Rev. 80, 939 (1950).

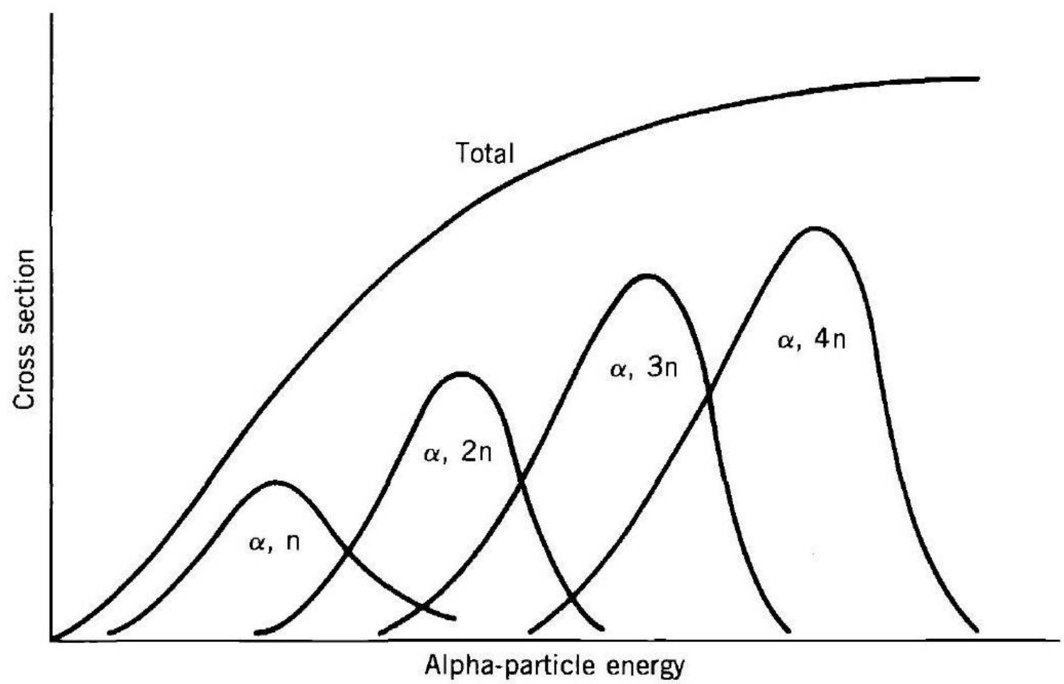


The curve marked NC shows the contribution from compoundnucleus formation to the cross section of the reaction $^{25}\mathrm{Mg(p,p)}^{25}\mathrm{Mg}$. The curve marked ID shows the contribution from direct reactions. Note that the direct part has a strong angular dependence, while the compound-nucleus part shows little angular dependence. From A. Gallmann et al., Nucl. Phys. 88, 654 (1966).

If this model were correct, we would expect for example that the relative cross sections for $^{63}\mathrm{Cu(p,n)}\,^{63}\mathrm{Zn}$ and $^{60}\mathrm{Ni}(\alpha,\mathrm{n})\,^{63}\mathrm{Zn}$ would be the same at incident energies that give the same excitation energy to $^{64}\mathrm{Zn}^{*}$. Figure shows the cross sections for the three final states, with the energy scales for the incident protons and α 's shifted so that they correspond to a common excitation of the compound nucleus. The agreement between the three pairs of cross sections is remarkably good, showing that indeed, the decay of $^{64}\mathrm{Zn}^{*}$ into any specific final state is nearly independent of how it was originally formed.

The compound-nucleus model works best for low incident energies ($10-20$ MeV), where the incident projectile has a small chance of escaping from the nucleus with its identity and most of its energy intact. It also works best for medium-weight and heavy nuclei, where the nuclear interior is large enough to absorb the incident energy.

Another characteristic of compound-nucleus reactions is the angular distribution of the products. Because of the random interactions among the nucleons, we expect the outgoing particle to be emitted with a nearly isotropic angular distribution (that is, the same in all directions). This expectation is quite consistent with experiment, as shown in Figure. In cases in which a heavy ion is the incident particle, large amounts of angular momentum can be transferred to the compound nucleus, and to extract that angular momentum the



At higher incident energies, it is more likely that additional neutrons will "evaporate" from the compound nucleus. emitted particles tend to be emitted at right angles to the angular momentum, and thus preferentially at 0 and 180°. With light projectiles, this effect is negligible.

The "evaporation" analogy mentioned previously is really quite appropriate. In fact, the more energy we give to the compound nucleus, the more particles are likely to evaporate. For each final state, the cross section has the Gaussian-like shape shown in Figure 11.19. Figure 11.21 shows the cross sections for (α, xn) reactions, where $x = 1,2,3, \ldots$. For each reaction, the cross section increases to a maximum and then decreases as the higher energy makes it more likely for an additional neutron to be emitted.

Reciprocity theorem

Let as consider a reversible process $X + x = Y + y$, in which X, x, Y and y occur in arbitrary numbers in a large box of volume V. We are interested in the relation between the total cross-section $\sigma(x \rightarrow y)$, most generally $\sigma\ (\alpha \rightarrow \beta)$ of the reaction with entrance channel α and reaction channel β and the total crosssection $\sigma(\beta \rightarrow \alpha)$ of the inverse reaction. For this we use the fundamental theorem of statistical mechanics (the principle of overall balance), which states that when the system is in dynamical equilibrium all energetically permissible states are occupied with equal probability. Here we are interested in two particular states, the reaction channels α and β. The theorem is then equivalent to starting the given energy range the number of possible channels in the box is proportional to the number of possible channels into the box. The latter is given by

$$N_\alpha = \frac{4\pi P_\alpha{}^2 V dp_\alpha}{h^3} = \frac{P_\alpha{}^2 V dp_\alpha}{2\pi^2 h^3}$$

Since v = dE/dp, hence $\quad N_\alpha = \frac{P_\alpha{}^2 V dE_\alpha}{2\pi^2 h^3 v_\alpha}.$

Similarly, we have $\quad N_\beta = \frac{P_\beta{}^2 V dE_\beta}{2\pi^2 h^3 v_\beta}.$

The energy range of the two channels must be the same, ie. $dE_\alpha = dE_\beta$ that is number of channels α in the box/ number of channels β in the box

$$\frac{N_\alpha}{N_\beta} = \frac{P_\alpha{}^2 V_\beta}{P_\beta{}^2 V_\alpha}$$

The system is in dynamical equilibrium when the number of $\alpha \rightarrow \beta$ transitions per second is equal to the number of $\beta \rightarrow \alpha$ transitions per second. The condition usually holds and is known as the principle of detailed balance. Further No. of transitions $\alpha \rightarrow \beta$ per sec $=$ $N_{\alpha} \times T(\alpha \rightarrow \beta)$,

Where $T(\alpha \rightarrow \beta)$ is the transition probability for the transition $\alpha \rightarrow \beta$. Hence $P_{\alpha}{}^{2}V_{\beta}T(\alpha \rightarrow \beta) = P_{\beta}{}^{2}V_{\alpha}T(\beta \rightarrow \alpha)$

The transition probability measures the chance that one particle moving with velocity v in volume V is scattered per sec. Hence the cross-section σ which corresponds to unit incident flux is given by the relation $\sigma = \mathrm{TV}/v$.

Combining relations and using $\mathrm{k} = \mathrm{p}/\hbar$, we have

$$k_{\alpha^2}\sigma(\alpha \rightarrow \beta) = k_{\beta^2}\sigma$$
$$\sigma(\alpha \rightarrow \beta)\lambda_{\alpha}^{2} = \sigma(\beta \rightarrow \alpha)/\lambda_{\beta}^{2}$$

We have assumed zero intrinsic angular moments for the particles so far. If $\mathbf{J}$ is the intrinsic angular momentum of any one of the particles, the corresponding density of states then must be multiplied by $2\,\mathrm{J} + 1$. Thus if there are intrinsic momenta for X, x, Y and y , we may write

$$(2\,\mathrm{J}_{\mathrm{X}} + 1)(2J_{x} + 1)\mathrm{k}_{\alpha}^{2}\sigma(\alpha \rightarrow \beta) = (2J_{Y} + 1)(2J_{y} + 1)\mathrm{k}_{\beta}^{2}\sigma(\beta \rightarrow \alpha)$$

If the initial and final states have definite angular momenta, then the above equation must be employed.

Resonance Reactions And Breit Weigner One Level Formula

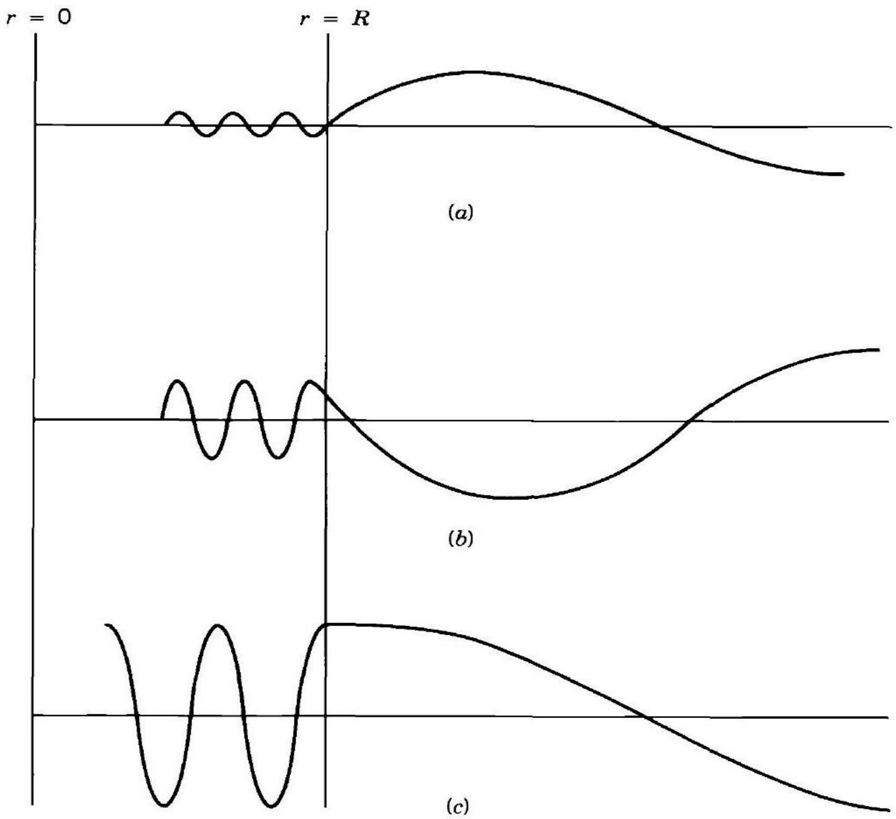
The compound-nucleus model of nuclear reactions treats the unbound nuclear states as if they formed a structureless continuum. That is, there may be discrete nuclear states, but there are so many of them and they are so close together that they form a continuous spectrum. Each of these supposed discrete states is unstable against decay and therefore has a certain width; when the states are so numerous that their spacing is much less than the widths of the individual states, the compound-nucleus continuum results.

The bound states studied by direct reactions are at the opposite end of the scale. Because they are stable against particle emission, their mean lives are much longer (for example, characteristic of γ decay) and their corresponding widths are much smaller. A state with a lifetime of 1 ps , for instance, has a width of about 10^{-3}eV, far smaller than the typical spacing of bound states. We are therefore justified in treating these as discrete states with definite wavefunctions.

Between these two extremes is the resonance region-discrete levels in the compound-nucleus region. These levels have a high probability of formation (large cross sections), and their widths are very small because at low incident energy, where these resonances are most likely to occur, the quasibound state that is formed usually has only two modes of decay available to it-re-ejecting the incident particle, as in elastic or inelastic scattering, or γ emission.

To obtain a qualitative understanding of the formation of resonances, we represent the nuclear potential seen by the captured particle as a square well. The oscillatory wave functions inside and outside the well must be matched smoothly, as we did for nucleon-nucleon scattering. Figure shows several examples of how this might occur. Depending on the phase of the wave function inside the nucleus, the smooth matching can result in substantial variations between the relative amplitudes of the wave functions inside and outside the nucleus. In case (a), the incident particle has relatively little probability to penetrate the nucleus and form a quasibound state; in case (c), there is a very high probability to penetrate. As we vary the energy of the incident particle, we vary the relative phase of the inner and outer wave functions; the location of the matching point and the relative amplitudes vary accordingly. Only for certain incident energies do we achieve the conditions shown in part (c) of Figure. These are the energies of the resonances in the cross section.

In a single, isolated resonance of energy E_{R} and width Γ, the energy profile of the cross section in the vicinity of the resonance will have the character of the energy distribution of any decaying state of lifetime $\tau=\hbar/\Gamma$; see, for example, Equation 6.20 or Figure 6.3. The resonance will occur where the total cross



(a) Far from resonance, the exterior and interior wave functions match badly, and little penetration of the nucleus occurs. (b) As the match improves, there is a higher probability to penetrate. (c) At resonance the amplitudes match exactly, the incident particle penetrates easily, and the cross section rises to a maximum. section has a maximum; assuming only one partial wave ℓ is important for the resonant state, there will be a scattering resonance where $\eta_\ell = -1$, corresponding to a phase shift $\delta_\ell = \pi/2$.

The shape of the resonance can be obtained by expanding the phase shift about the value $\delta_\ell = \pi/2$. Better convergence of the Taylor series expansion is obtained if we expand the cotangent of $\boldsymbol{\delta_\ell}$:

$$\cot\delta_\ell(E) = \cot\delta_\ell(E_\mathrm{R}) + (E - E_\mathrm{R})\left(\frac{\partial \cot\delta_\ell}{\partial E}\right)_{E=E_\mathrm{R}}$$

in which

$$\left(\frac{\partial \cot \delta_\ell}{\partial E}\right)_{E=E_\mathrm{R}} = -\left(\frac{\partial \delta_\ell}{\partial E}\right)_{E=E_\mathrm{R}}$$

Defining the width Γ as

$$\Gamma = 2\left(\frac{\partial \delta_\ell}{\partial E}\right)^{-1}_{E=E_\mathrm{R}}$$

then it can be shown that the second-order term vanishes, and thus (neglecting higher-order terms)

$$\cot \delta_\ell = -\frac{(E - E_\mathrm{R})}{\Gamma/2}$$

Because Γ is the full width of the resonance, the cross section should fall to half of the central value at $E - E_\mathrm{R} = \pm\Gamma/2$. From Equation 11.63, this occurs when $\cot \delta_\ell = \pm 1$, or $\delta_\ell = \pi/4, 3\pi/4$ (compared with $\delta_\ell = \pi/2$ at the center of the resonance). The cross section depends on $\sin^2 \delta_\ell$, which does indeed fall to half the central value at $\delta_\ell = \pi/4$ and $3\pi/4$.

we find

$$\sin \delta_\ell = \frac{\Gamma/2}{[(E - E_\mathrm{R})^2 + \Gamma^2/4]^{1/2}}$$

and the scattering cross section becomes,

$$\sigma_\mathrm{sc} = \frac{\pi}{k^2}(2\ell + 1)\frac{\Gamma^2}{(E - E_\mathrm{R})^2 + \Gamma^2/4}$$

This result can be generalized in two ways. In the first place, we can account for the effect of reacting particles with spin. If s_a and s_X are the spins of the incident and target particles, and if I is the total angular momentum of the resonance,

$$\boldsymbol{I} = s_\mathrm{a} + s_\mathrm{X} + \ell$$

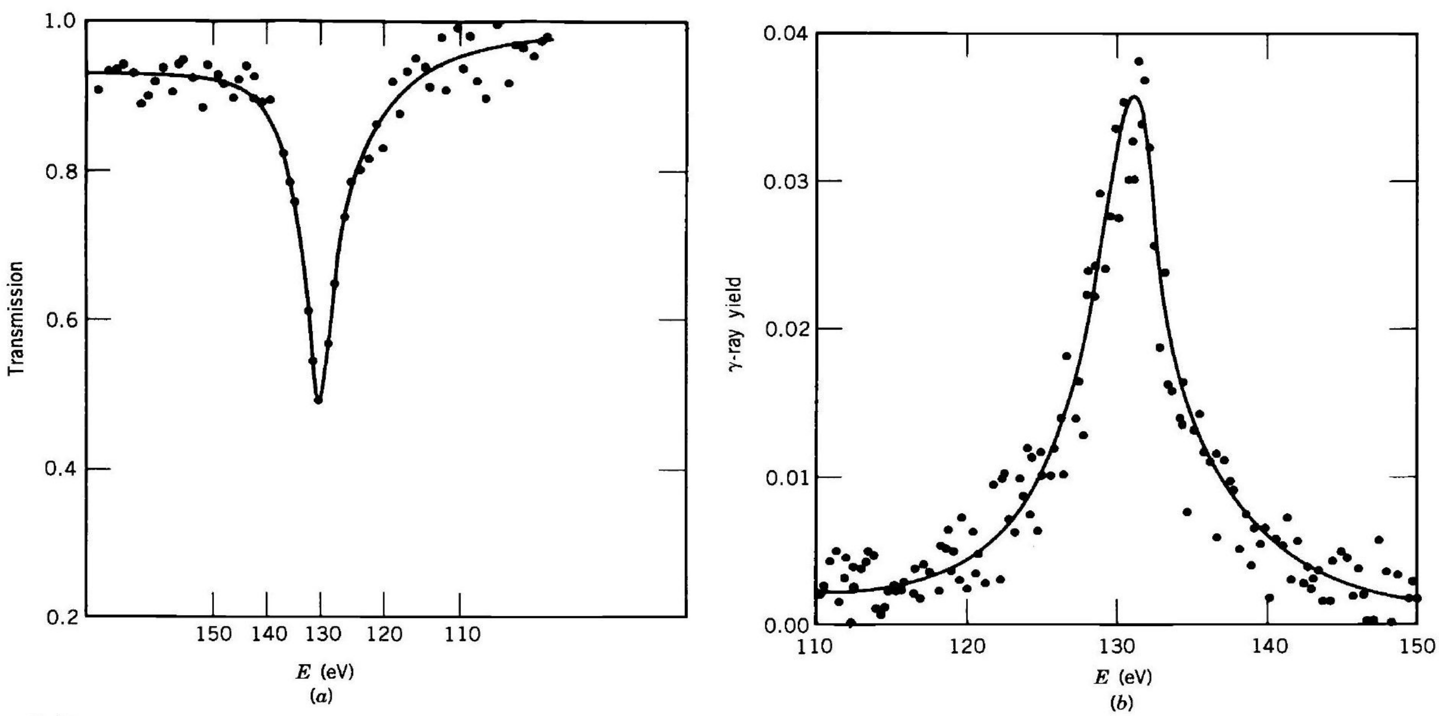
then the factor $(2\ell + 1)$ should be replaced by the more general statistical factor

$$g = \frac{2I + 1}{(2s_\mathrm{a} + 1)(2s_\mathrm{X} + 1)}$$

Note that g reduces to $(2\ell+1)$ for spinless particles. The second change we must make is to allow for partial entrance and exit widths. If the resonance has many ways to decay, then the total width Γ is the sum of all the partial widths Γ_i

$$\Gamma = \sum_i \Gamma_i$$

The Γ^2 factor in the denominator is related to the decay width of the resonant state and therefore to its lifetime: $\Gamma = \hbar/\tau$. The observation of only a single entrance or exit channel does not affect this factor, for the resonance always decays with the same lifetime τ. In the analogous situation in



130 − eV neutron resonance in scattering from ^{59}Co. Part (a) shows the intensity of neutrons transmitted through a target of ^{59}Co; at the resonance there is the highest probability for a reaction and the intensity of the transmitted beam drops to a minimum. In (b), the γ-ray yield is shown for neutron radiative capture by ^{59}Co. Here the yield of γ rays is maximum where the reaction has the largest probability. From J. E. Lynn, The Theory of Neutron Resonance Reactions (Oxford: Clarendon, 1968).

radioactive decay, the activity decays with time according to the total decay constant, even though we might observe only a single branch with a very different partial decay constant. The Γ^2 factor in the numerator, on the other hand, is directly related to the formation of the resonance and to its probability to decay into a particular exit channel. In the case of elastic scattering, for which Equation 11.65 was derived, the entrance and exit channels are identical. That is, for the reaction $a + X \rightarrow a + X$, we should use the partial widths Γ_a of the entrance and exit channels:

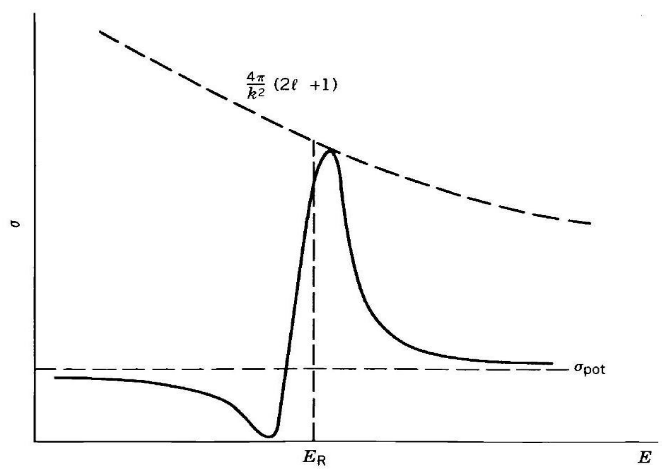
$$\sigma = \frac{\pi}{k^2} g \frac{(\Gamma_{ax})^2}{(E - E_R)^2 + \Gamma^2/4}$$

Similarly, for the reaction $a + X \rightarrow b + Y$, a different exit width must be used:

$$\sigma = \frac{\pi}{k^2} g \frac{\Gamma_{aX}\Gamma_{bY}}{(E - E_R)^2 + \Gamma^2/4}$$

Above equations are examples of the Breit-Wigner formula for the shape of a single, isolated resonance. Figure shows such a resonance with the Breit-Wigner shape. The cross section for resonant absorption of γ radiation has a similar shape.

Many elastic scattering resonances have shapes slightly different from that suggested by the Breit-Wigner formula. This originates with another contribution to the reaction amplitude from direct scattering of the incident particle by the nuclear potential, without forming the resonant state. This alternative process is called potential scattering or shape-elastic scattering. Potential scattering and

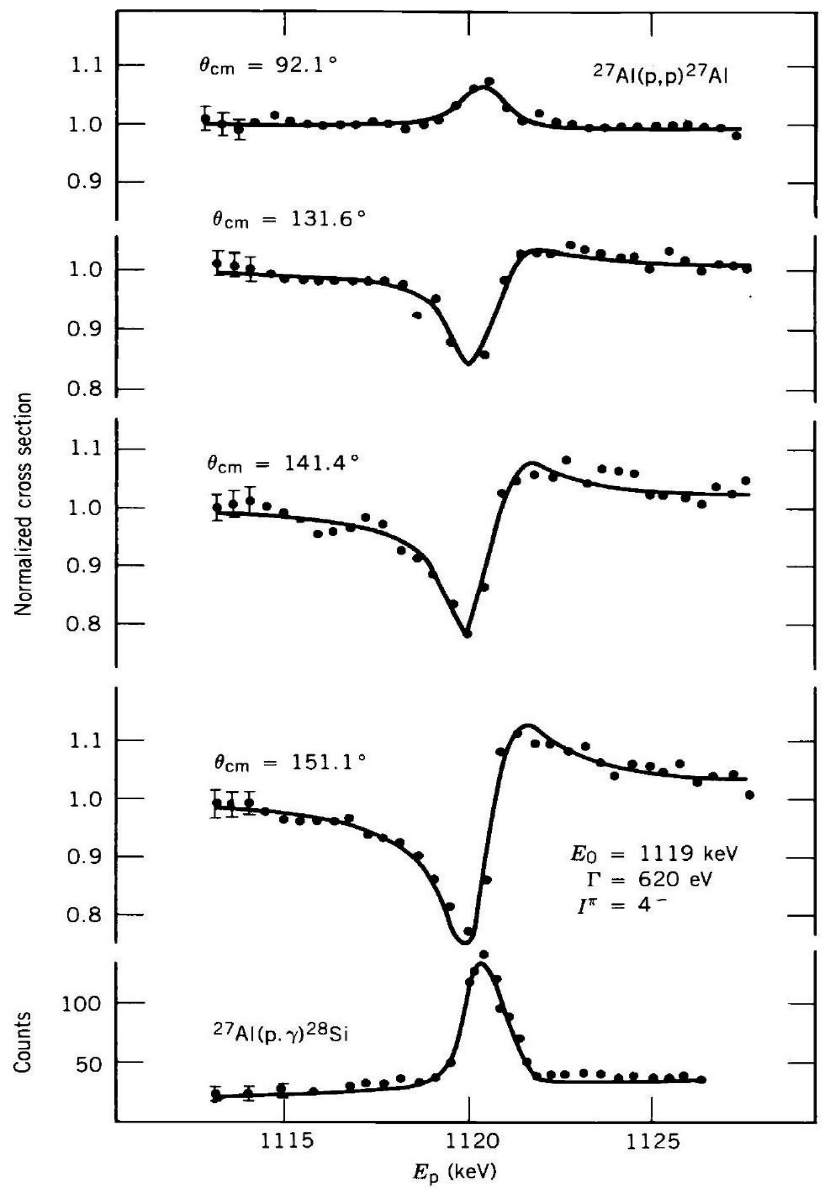


Interference between resonance and potential scattering produces resonances with this characteristic shape.

resonant scattering both contribute to the elastic scattering amplitude, and interference between the two processes causes variation in the cross section. Interference can cause the combined cross section to be smaller than it would be for either process alone. It is therefore not correct simply to add the cross sections for the two processes. We can account for the two processes by writing

$$\eta_\ell = e^{2i(\delta_{\ell R}+\delta_{\ell P})}$$

where $\delta_{\ell R}$ is the resonant phase shift, as in Equations 11.63 or 11.64 , and $\delta_{\ell P}$ is an additional contribution to the phase shift from potential scattering. From



Resonances in the reaction $^{27}\mathrm{Al}(\mathrm{p},\mathrm{p})^{27}\mathrm{Al}$. The resonances occur in the nucleus $^{28}\mathrm{Si}$. Note that the (p,γ) yield shows a resonance at the same energy. From A. Tveter, Nucl. Phys. A 185, 433 (1972).

Direct reactions

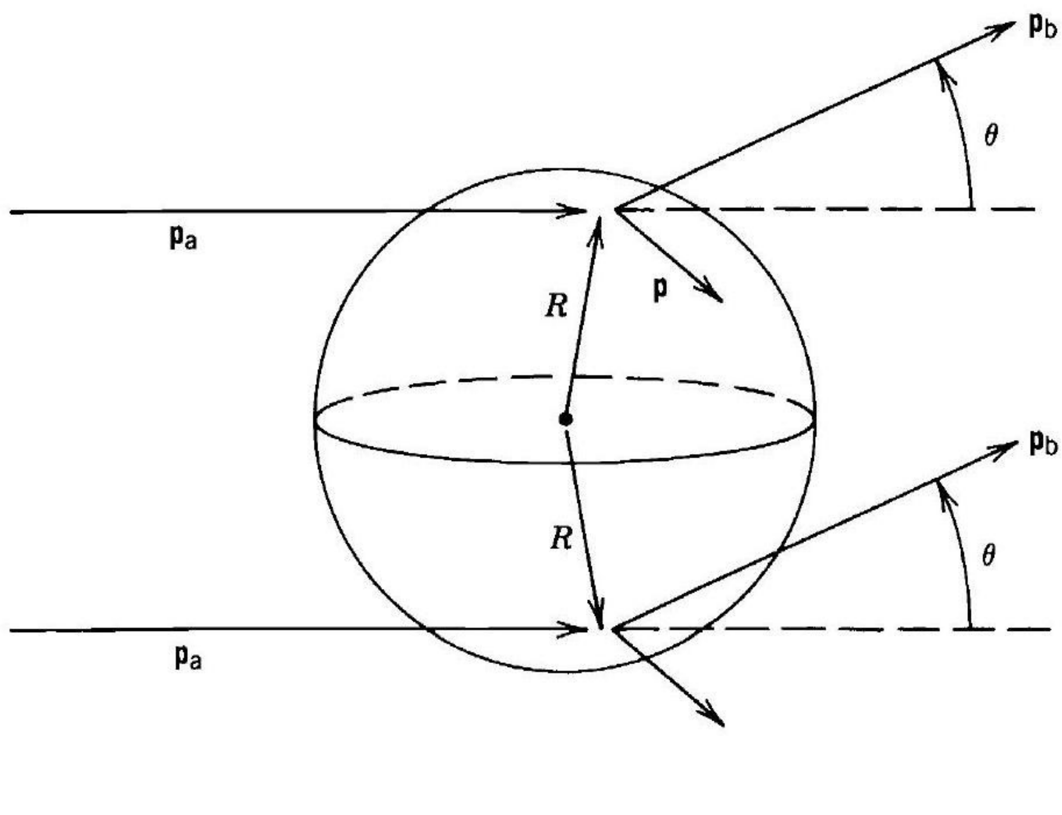
At the opposite extreme from compound-nucleus reactions are direct reactions, in which the incident particle interacts primarily at the surface of the target nucleus; such reactions are also called peripheral processes. As the energy of the incident particle is increased, its de Broglie wavelength decreases, until it becomes more likely to interact with a nucleon-sized object than with a nucleus-sized object. A $1-\mathrm{MeV}$ incident nucleon has a de Broglie wavelength of about 4 fm , and thus does not "see" individual nucleons; it is more likely to interact through a compound-nucleus reaction. A $20-\mathrm{MeV}$ nucleon has a de Broglie wavelength of about 1 fm and therefore may be able to participate in direct processes. Direct processes are most likely to involve one nucleon or very few valence nucleons near the surface of the target nucleus.

Of course, it may be possible to have direct and compound-nucleus processes both contribute to a given reaction. How can we distinguish their contributions or decide which may be more important? There are two principal differences that can be observed experimentally: (1) Direct processes occur very rapidly, in a time of the order of 10^{-22} s, while compound-nuclear processes typically take much longer, perhaps 10^{-16} to 10^{-18} s. This additional time is necessary for the distribution and reconcentration of the incident energy. There are ingenious experimental techniques for distinguishing between these two incredibly short intervals of time. (2) The angular distributions of the outgoing particles in direct reactions tend to be more sharply peaked than in the case of compound-nuclear reactions.

Inelastic scattering could proceed either through a direct process or a compound nucleus, largely depending on the energy of the incident particle. The deuteron stripping reaction (d,n), which is an example of a transfer reaction in which a single proton is transferred from projectile to target, may also go by either mechanism. Another deuteron stripping reaction (d,p) may be more likely to go by a direct process, for the "evaporation" of protons from the compound nucleus is inhibited by the Coulomb barrier. The (α,n) reaction is less likely to be a direct process, for it would involve a single transfer of three nucleons into valence states of the target, a highly improbable process.

One particularly important application of single-particle transfer reactions, especially (d, p) and (d, n), is the study of low-lying shell-model excited states. Several such states may be populated in a given reaction; we can choose a particular excited state from the energy of the outgoing nucleon. Once we have done so, we would like to determine just which shell-model state it is. For this we need the angular distribution of the emitted particles, which often give the spin and parity of the state that is populated in a particular reaction. Angular distributions therefore are of critical importance in studies of transfer reactions. (Pickup reactions, for example (p, d), in which the projectile takes a nucleon from the target, also give information on single-particle states.)

Let's consider in somewhat more detail the angular momentum transfer in a deuteron stripping reaction. In the geometry of Figure 11.22, an incident particle with momentum $\boldsymbol{p}_{\mathrm{a}}$ gives an outgoing particle with momentum $\boldsymbol{p}_{\mathrm{b}}$, while the residual nucleus (target nucleus plus transferred nucleon) must recoil with momentum $\boldsymbol{p} = \boldsymbol{p}_{\mathrm{a}} - \boldsymbol{p}_{\mathrm{b}}$. In a direct process, we may assume that the transferred nucleon instantaneously has the recoil momentum and that it must be placed in an orbit with orbital angular momentum $\ell = Rp$, assuming that the interaction



Geometry for direct reactions occurring primarily on the nuclear surface. takes place at the surface of the nucleus. The momentum vectors are related by the law of cosines:

$$p^2 = p_{\mathrm{a}}^2 + p_{\mathrm{b}}^2 - 2p_{\mathrm{a}}p_{\mathrm{b}}\cos\theta$$

Given the energies of the incident and outgoing particles, we then have a direct relationship between ℓ and θ-particles emerging at a given angle should correspond to a specific angular momentum of the orbiting particle.

Consider a specific example, the (d, p) reaction on $^{90}\mathrm{Zr}$ leading to single neutron shell-model states in $^{91}\mathrm{Zr}$. The Q value is about 5 MeV , so an incident deuteron at 5 MeV gives a proton at about 10 MeV , less any excitation in $^{91}\mathrm{Zr}$. Since at these energies $p_{\mathrm{a}} \simeq p_{\mathrm{b}} \simeq 140 \mathrm{MeV}/c$.

$$\ell = \left[\frac{2c^2 p_{\mathrm{a}} p_{\mathrm{b}} (2\sin^2\theta/2)}{\hbar^2 c^2/R^2}\right]^{1/2} \cong 8\sin\frac{\theta}{2}$$

For each angular momentum transfer, we expect to find outgoing protons at the following angles: $\ell = 0, 0°; \ell = 1, 14°; \ell = 2, 29°; \ell = 3, 44°$.

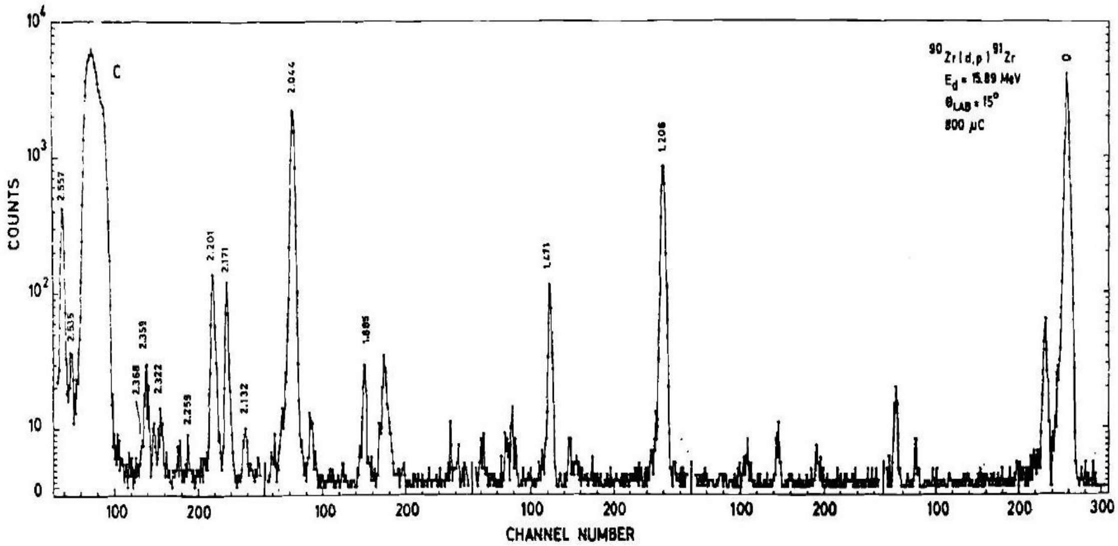
This simple semiclassical estimate will be changed by the intrinsic spins of the particles, which we neglected. There will also be interference between scatterings that occur on opposite sides of the nucleus, as shown in Figure. These interferences result in maxima and minima in the angular distributions. Figure shows the result of studies of (d, p) reactions on $^{90}\mathrm{Zr}$. You can see several low-lying states in the proton spectrum, and from their angular distributions we can assign them to specific spins and parities in $^{91}\mathrm{Zr}$. Notice the appearance of maxima and minima in the angular distribution.

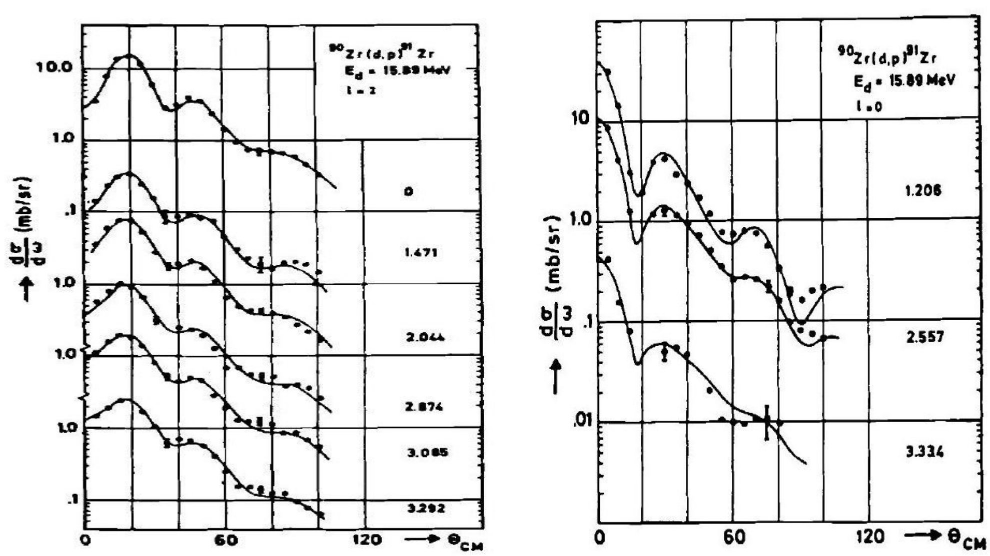
The angular momentum transfer, as usual, also gives us the change in parity of the reactions, $\ell =$ even for no change in parity and $\ell =$ odd for a change in parity. If we are studying shell-model states in odd- A nuclei by single-particle transfer reactions such as (d, p), we will use an even- Z, even $-N$ nucleus as target, and so the initial spin and parity are 0^+. If the orbital angular momentum transferred is ℓ, then the final nuclear state reached will be $\ell \pm \frac{1}{2}$, allowing for the contribution of the spin of the transferred nucleon. For $\ell = 2$, for instance, we can reach states of $j = \frac{3}{2}$ or $\frac{5}{2}$, both with even parity.

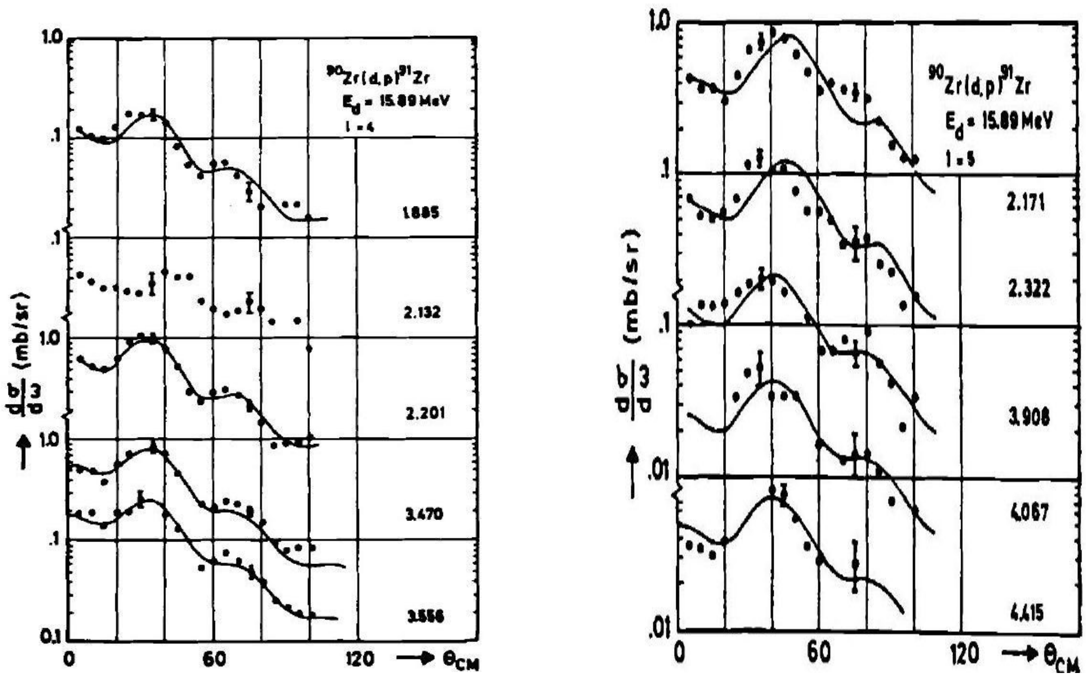
The complete theory of direct reactions is far too detailed for this text, but we can sketch the outline of the calculation as an exercise in applications of the principles of quantum mechanics. The transition amplitude for the system to go from the initial state $(\mathrm{X}+\mathrm{a})$ to the final state $(\mathrm{Y}+\mathrm{b})$ is governed by the usual quantum mechanical matrix element:

$$M=\int \psi_{\mathrm{y}}^{*}\psi_{\mathrm{b}}^{*}V\psi_{\mathrm{x}}\psi_{\mathrm{a}}dv$$

The interaction V must be a very complicated function of many nuclear coordinates. A simplifying assumption is the plane-wave Born approximation, in which ψ_{a} and ψ_{b} are treated as plane waves. Expanding the resulting exponential $e^{i\boldsymbol{p}\cdot\mathbf{r}/\hbar}$ using a spherical wave expansion of the form of Equation 11.31 and making the simplifying assumption that the interaction takes place on the nuclear

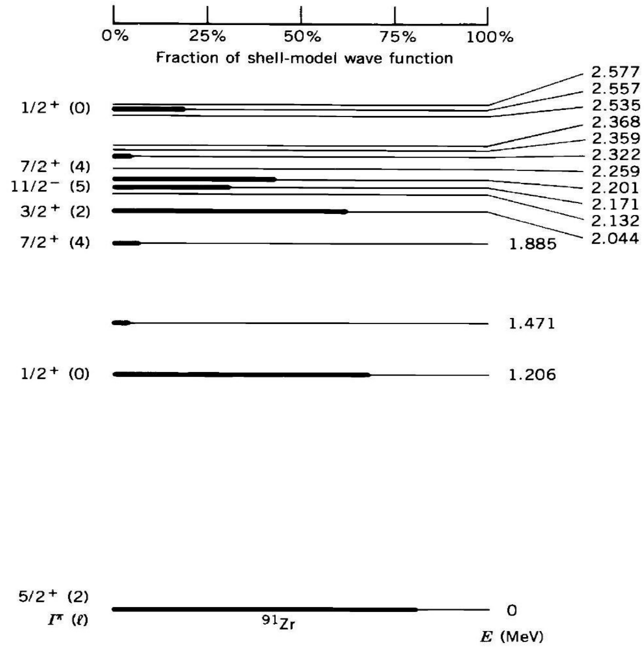






Proton spectrum from $^{90}\mathrm{Zr(d,p)}^{91}\mathrm{Zr}$. Peaks are identified with the final states in $^{91}\mathrm{Zr}$ populated. The large peak at the left is from a carbon impurity. (bottom) Angular distributions fitted to determine the ℓ value. Note that the location of the first maximum shifts to larger angles with increasing ℓ, as predicted by Equation 11.57. See Figure 11.24 for the deduced excited states. Data from H. P. Blok et al., Nucl. Phys. A 273, 142 (1976).

surface, so the integral is evaluated only at $r = R$, the matrix element is proportional to $j_\ell(kR)$ where $k = p/\hbar$ contains the explicit angular dependence through the Equation. The cross section then depends on $[j_\ell(kR)]^2$, which gives results of the form of Figure. Taking this calculation one step further, we use the optical model to account for the fact that the incoming and outgoing plane waves are changed (or distorted) by the nucleus. This gives the distorted-wave Born approximation, or DWBA. We can even put in explicit shell-model wave functions for the final state, and ultimately, we find a differential cross section for the reaction. Because there are no "pure" shell-model states, the calculated cross section may describe many different final states. Each will have a differential cross section whose shape can be accurately calculated based on this model, but the amplitude of the cross section for any particular state depends on the fraction of the pure shell-model state included in the wave function for that state. The measured cross section is thus reduced from the calculated shell-model single-particle value by a number



Deduced level scheme for $^{91}\mathrm{Zr}$. Each ℓ value (except zero) deduced from the angular distributions of Figure 11.23 leads to a definite parity assignment but to two possible / values, $\ell \pm \frac{1}{2}$. Which one is correct must be determined from other experiments. The fraction of the single-particle strengths represented by each level is indicated by the length of the shading; thus the ground state is nearly pure $d_{5/2}$ shell-model state.

between 0 and 1 called the spectroscopic factor S :

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{meas}} = S\left(\frac{d\sigma}{d\Omega}\right)_{\text{calc}}$$

A pure shell-model state would have $S = 1$. In practice we often find the shell-model wave function to be distributed over many states. Figure 11.24 shows examples of the spectroscopic factors measured for $^{91}\mathrm{Zr}$.

Nuclear chain reaction

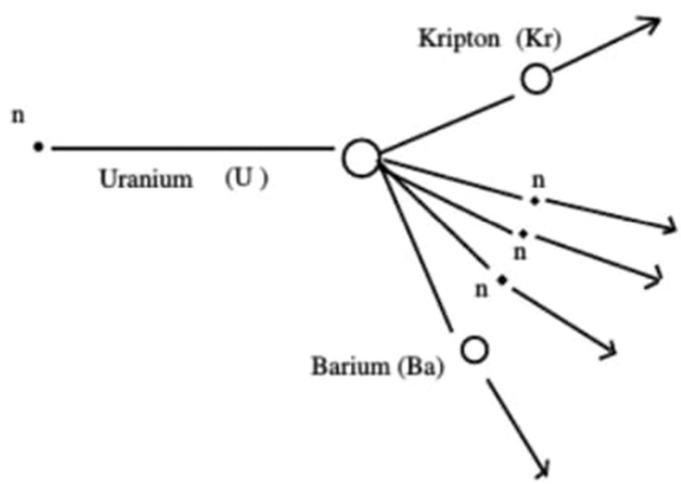
The schematic equation for the fission process is

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236} * \rightarrow X + Y + \text{ neutrons.}$$

$_{92}U^{236*}$ is a highly unstable isotope of uranium, and X and Y are the fission fragments. The fragments are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are given off. Typical fission reactions are

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236*} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3n^{1} + Q$$
$$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236} * \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_{0}n^{1} + Q$$

Where Q is the energy released in the reaction. According to equation (2) $_{92}U^{235}$ is bombarded by slow moving neuron, the nucleus becomes unstable ($_{92}U^{236}$ $*$) and splits into $_{56}Ba^{141}$ and Where Q is the energy released in the reaction. According to equation (2) $_{92}U^{235}$ is bombarded by slow moving neuron, the nucleus becomes unstable ($_{92}U^{236}$ $*$) and splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons and energy Q. The number of 2 neutrons are released. So, in each fission average of 2.5 neutrons are released.

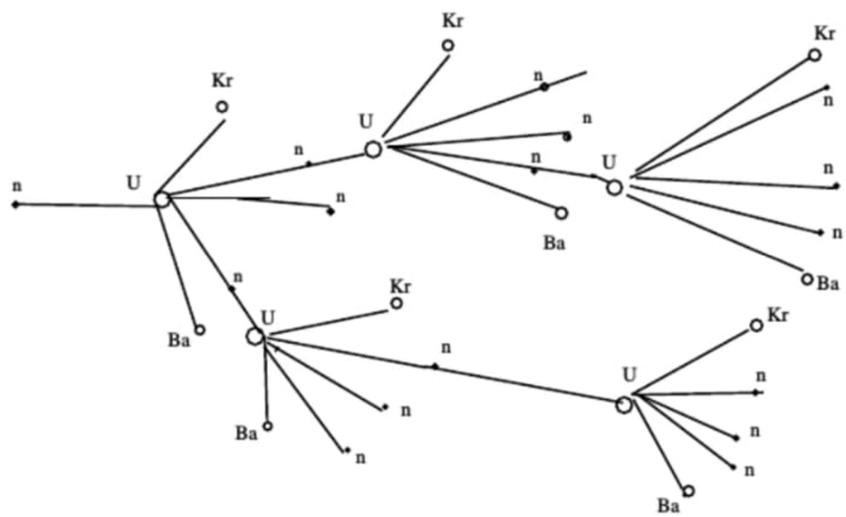


Schematic representation of nuclear fission

In this fission process 200 MeV energy is released. These three neutrons are capable of splitting another three uranium nuclei and thus releases 9 neutrons. Like this the process spread out, and the number of neutrons and the amount of energy increases at a rapid rate. Such a self-propagation process is called Chain reaction.

There are two types of chain reaction. They are controlled and uncontrolled reactions. In the controlled chain reaction, the neutrons are built up certain level and there after the number of fissions producing neutrons are kept constant. This principle is used in Nuclear reactors.

In the uncontrolled nuclear reaction the fission producing neutrons is allowed to multiply indefinitely and the entire energy releases all at once. This type of reaction takes place in Atom bomb



Nuclear Chain Reaction

Suppose a single neutron causing fission in a uranium nucleus produces 3 prompt neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing nine neutrons. These 9 neutrons in turn to cause fission in nine uranium nuclei producing 27 neutrons. And so on. The number of neutrons produced in n such generations is 3^n neutrons. The ratio of secondary neutrons produced to the original neutrons is called the multiplication factor (k).

Consider 1 kg of $^{235}U_{92}$ which contains $6.023 \times 10^{26}/235$ or about 25×10^{23} atoms. Suppose a stray of neutron causes fission in a uranium nucleus. Each fission will release on the average 2.5 neutrons. The velocity of the neutron among the uranium atoms is such that a fission capture of thermal neutron by the $_{92}U^{235}$ nuclei take place in about 10^{-8} s each of this fission, in turn, release 2.5 neutrons. Let us assume that all these neutrons are available for inducing further fission reactions. Let n be the number of stages of fission captures required to disrupt the entire mass of 1 kg of $^{235}U_{92}$. Then

$$(2.5)^{n} = 25 \times 10^{23} \text{ or } n = 60.$$

The time required for 60 fissions to take place $= 60 \times 10^{-8}$ s $= 0.6\mu$ s. Since each fission releases about 200 MeV of energy, this means that a total of $200 \times 25 \times 10^{23} = 5 \times 10^{26}$ MeV of energy is released in 0.6μ s.

The release of this tremendous amount of energy in such a short time interval leads to a violent explosion. This results in powerful air blasts and high temperature of the order of 10^{7} K or more, besides intense radioactivity.

Critical Mass for maintenance of chain reaction: Consider a system consisting of uranium (as fissile material) and a moderator. Even though each neutron that produces fission ejects 2.5 neutrons on an average, all of them are not available for further fission. The maintenance of chain reaction depends upon a favorable balance of neutrons among the three processes given below: i) The fission of uranium nuclei which produces more neutrons than the number of neutrons used for inducing fission.

$$\frac{\text{escape rate}}{\text{production rate}} = \frac{1}{r}$$

The larger the size of the body, the smaller is the escape rate. Thus, it is clear that by increasing the volume of the system, the loss of neutrons by escape from the system is reduced. The greater the size of the system, the lesser is the possibility of the escape of neutrons. In this case, the production of neutrons will be more than the loss due to other causes and a chain reaction can be maintained. Thus, there is a critical size for the system. Critical size of a system containing fissile material is define as the minimum size for which the number of neutrons produced in the fission process just balance those lost leakage and non-fission capture. The mass of the material at this size is called the critical mass. If the size is less than the critical size, a chain reaction is not possible.

Four factor formula

Let's consider an infinitely large mass of uranium, which we will for the present assume to be of natural isotopic composition ($0.72\%\ ^{235}U, 99.28\%\ ^{238}U$). A single fission event will produce, on the average, about 2.5 neutrons. Each of these "second-generation" neutrons is capable of producing yet another fission event producing still more neutrons, and so on. This process is the chain reaction. Each fission event releases about 200 MeV in the form of kinetic energy of heavy fragments (that is, heat) and radiation.

It is convenient to define the neutron reproduction factor k_∞ (for an infinite medium, that is, ignoring loss of neutrons through leakage at the surface). The reproduction factor gives the net change in the number of thermal neutrons from one generation to the next; on the average, each thermal neutron produces k_∞ new thermal neutrons. For a chain reaction to continue, we must have $k_\infty \geq 1$. Although we have an average of 2.5 neutrons emitted per fission, these are fast neutrons, for which the fission cross section is small. It is advantageous to moderate these neutrons to thermal velocities because of the large thermal cross section (about 580 b) resulting from the $1/v$ law. In the process, many neutrons can become absorbed or otherwise lost to the chain reaction, and the 2.5 fast neutrons per fission can easily become < 1 thermal neutron, effectively halting the reaction.

As we discussed in Section 12.2, neutrons lose energy in elastic collisions with nuclei, and a popular choice for a moderator is carbon in the form of graphite blocks. (The best choice for a moderator is the lightest nucleus, to which the neutron transfers the largest possible energy in an elastic collision. Carbon is a reasonable choice because it is available as a solid, thus with a high density of scattering atoms, and is inexpensive and easy to handle.) A lattice of blocks of uranium alternating with graphite is called a chain-reacting pile, and the first such pile was constructed by Fermi and his collaborators in a squash court at the University of Chicago in 1942. If the reproduction factor k (for a finite pile) is exactly 1.0, the pile is said to be critical; for $k < 1$, the pile is subcritical and for $k > 1$ it is supercritical. To maintain a steady release of energy, we would like for the pile to be exactly critical.

To calculate the reproduction factor k_∞, we must follow the fate of a collection of thermal neutrons from one generation to the next. Let's assume we have N thermal neutrons in the present generation. Even though each fission produces on the average ν neutrons, we will not have νN fast fission neutrons immediately available, for not every one of the original collections will cause a fission event. Some will be absorbed through other processes, most notably (n, γ) reactions in both $^{235}\mathrm{U}$ and $^{238}\mathrm{U}$. We define η as the mean number of fission neutrons produced per original thermal neutron. It is clear that $\eta < \nu$, for some of the original thermal neutrons do not cause fissions. If we let σ_{f} represent the fission cross section and σ_a the cross section for other absorptive processes (both of these cross sections are evaluated for thermal neutrons), then the relative probability for a neutron to cause fission is $\sigma_\mathrm{f}/(\sigma_\mathrm{f}+\sigma_\mathrm{a})$ and

$$\eta = \nu \frac{\sigma_\mathrm{f}}{\sigma_\mathrm{f}+\sigma_\mathrm{a}}$$

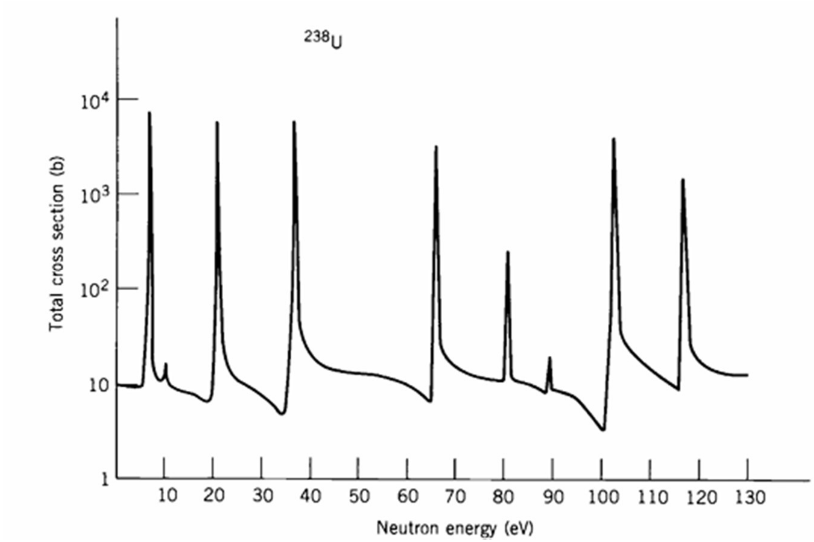
For $^{235}\mathrm{U}, \sigma_\mathrm{f} = 584$ b and $\sigma_\mathrm{a} = 97$ b, so that $\eta = 2.08$ fast neutrons are produced per thermal neutron. $^{238}\mathrm{U}$ is not fissionable with thermal neutrons, and so $\sigma_\mathrm{f} = 0$, but $\sigma_\mathrm{a} = 2.75$ b. For a natural mixture of $^{235}\mathrm{U}$ and $^{238}\mathrm{U}$, the effective fission and absorption cross sections are

$$\begin{aligned}\sigma_\mathrm{f} &= \frac{0.72}{100}\sigma_\mathrm{f}(235) + \frac{99.28}{100}\sigma_\mathrm{f}(238)\\ &= 4.20 \text{ b}\\ \sigma_\mathrm{a} &= \frac{0.72}{100}\sigma_\mathrm{a}(235) + \frac{99.28}{100}\sigma_\mathrm{a}(238)\\ &= 3.43 \text{ b}\end{aligned}$$

and the effective value of η becomes 1.33 . This is already very close to 1.0 , so we must minimize other ways that neutrons can be lost, in order to obtain a critical reactor. If we use enriched uranium, in which the fraction of $^{235}\mathrm{U}$ is raised to 3%, the effective value of η becomes 1.84 , considerably further from the critical value and allowing more neutrons to be lost by other means and still maintain the criticality condition.

At this point the N thermal neutrons have been partly absorbed and the remainder have caused fissions, and we now have ηN fast neutrons which must be reduced to thermal energies. As these fast neutrons journey through the chain-reacting pile, some of them may encounter $^{238}\mathrm{U}$ nuclei which have a small cross section (about 1 b) for fission by fast neutrons. This causes a small increase in the number of neutrons, so we introduce a new factor ε, the fast fission factor; the number of fast neutrons is now $\eta\varepsilon N$. The value of ε is about 1.03 for natural uranium.

Moderation of the neutrons is accomplished by intermixing the reactor fuel with a light moderator, such as carbon, usually in the form of graphite. It takes about 100 collisions with carbon for MeV neutrons to be thermalized. In the process, they must pass through the region from $10-100\mathrm{eV}$, in which $^{238}\mathrm{U}$ has many capture resonances with cross sections in the range of 1000 b (greater than the $^{235}\mathrm{U}$ fission cross section). If we are to have any thermal neutrons at all, we must find a way for the neutrons to avoid resonance capture. If the uranium and the graphite are intimately mixed, such as in the form of fine powders, it will be almost impossible for neutrons to avoid resonant capture in ^{238}U. In this kind of mixture, a neutron may scatter



Neutron-capture resonance region of ^{238}U.

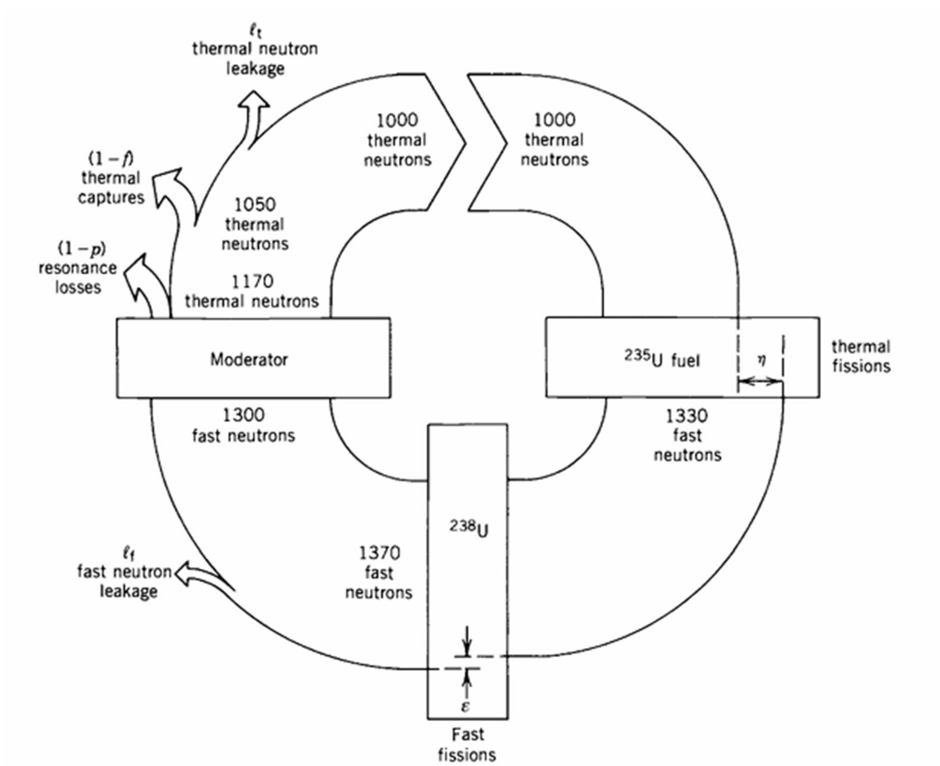
very few times from carbon before encountering a ^{238}U nucleus, and therefore will very likely pass near ^{238}U when its energy is in the critical region. If we make the lumps of graphite larger, we will eventually reach a configuration in which the neutrons can be completely thermalized through many scatterings without leaving the graphite and encountering any ^{238}U. In this way it is possible to avoid the dangerous resonance region. The average distance needed by a fission neutron to slow to thermal energies in graphite is about 19 cm . If we therefore construct the pile as a matrix of uranium fuel elements separated by about 19 cm of graphite, we minimize the neutron losses due to resonant capture. Of course, it will still be possible for some neutrons to wander too close to the surface of the graphite and enter the uranium before they are fully thermalized, and thus resonant capture cannot be completely eliminated. We account for this effect by including a reduction factor p, the resonance escape probability, in the number of neutrons remaining after thermalization, now $\eta \varepsilon p N$. A typical value of p might be 0.9 .

Once the neutrons are successfully thermalized, we must immediately get them into the uranium, so the graphite lumps mustn't be made too large. In any case, there is a probability of capture of thermal neutrons by graphite and by any structural components of the reactor (such as the material used to encapsulate the fuel elements). One reason for choosing carbon as a moderator is that it has a very small thermal cross section (0.0034 b), but there is a lot of it present. The thermal utilization factor f gives the fraction of the thermal neutrons that are actually available to ^{235}U and ^{238}U. This factor may also typically be about 0.9 .

The number of neutrons which finally survive capture by the moderator and other materials is $\eta\varepsilon p f N$, and whether this is greater or smaller than the original number N determines the criticality of the reactor. The reproduction factor is

$$k_\infty = \eta\varepsilon p f$$

which is known for obvious reasons as the four-factor formula. Figure 13.24 shows the processes that can occur to neutrons during a reactor cycle.



Schematic representation of processes occurring during a single generation of neutrons. The cycle has been drawn for a reproduction factor k of exactly 1.000.

The actual design of the pile will be a compromise of attempts to optimize the three factors that depend on the geometry (ε, p, f). For instance, large lumps of U tend to reduce p because resonant absorption occurs primarily on the surface; that is, a neutron of energy $10 - 100$eV that enters a uranium lump is unlikely to get very far before absorption. The uranium in the center of the lump never sees such neutrons, and that central mass of uranium can be treated from the perspective of resonant absorption as if it were not present. The larger the lumps of uranium, the more effective is the surface in shielding the central uranium from neutron absorption. On the other hand, if the lumps become too large, the same effect occurs for the thermal neutrons that cause fission-more fission occurs near the surface and the density of thermal neutrons decreases toward the center of the lump of uranium.

Using the four factors estimated for the natural uranium and graphite pile, the reproduction factor is estimated to be $k_\infty = 1.11$. This estimate is still not appropriate for an actual pile, for we have ignored leakage of neutrons at the surface. This leakage must be considered both for fast neutrons and for thermalized neutrons. If ℓ_t and ℓ_t are the fractions of each that are lost, the complete formula for the reproduction factor is

$$k = \eta \varepsilon p f (1 - \ell_\mathrm{f})(1 - \ell_\mathrm{t})$$

The larger the pile, the smaller is the surface area to volume ratio and the smaller is the fraction of neutrons that leak. If ℓ_f and ℓ_t are small, then $k_\infty - k \approx k(\ell_\mathrm{f} + \ell_\mathrm{t})$. We expect that the total leakage $(\ell_\mathrm{f} + \ell_\mathrm{t})$ decreases as the surface area increases. Furthermore, the leakage must increase with the distance a neutron is able to travel before absorption. This distance is called the migration length M and it includes two contributions: the diffusion length L_d for thermal neutrons, the distance a thermal neutron can travel on the average before absorption, and the slowing distance L_s over which a fast neutron slows to thermal energies:

$$M = \left(L_\mathrm{d}^2 + L_\mathrm{s}^2\right)^{1/2}$$

For graphite, $L_\mathrm{s} = 18.7$ cm and $L_\mathrm{d} = 50.8$ cm. If the pile has dimension R (radius, if it is spherical, or length of a side if a cube), it is reasonable to suppose that $(k_\infty - k) \propto R^{-2}$ and also that $k_\infty - k$ depends on M; if these are the only physical parameters involved, then from a dimensional analysis we expect

$$k_\infty - k \propto \frac{M^2}{R^2}$$

and there will be a critical size R_c corresponding to $k = 1$

$$R_\mathrm{c} \propto \frac{M}{\sqrt{k_\infty - 1}}$$

The proportionality constant needed to make Equation 13.10 an equality depends on the geometry but is of order unity. For a spherical arrangement,

$$R_{\mathrm{c}}=\frac{\pi M}{\sqrt{k_{\infty}-1}}$$

and our estimates for the natural uranium-graphite reactor give $R_{\mathrm{c}}=5$ m. A spherical lattice of radius 5 m consisting of uranium and graphite blocks should "go critical." The size can be decreased somewhat by surrounding the pile with a material that reflects escaping neutrons back into the pile.

Before leaving this brief introduction to reactor theory, let's consider the time constants involved in neutron multiplication. The neutrons are characterized by a time constant τ, which includes the time necessary to moderate (about 10^{-6} s) and a diffusion time at thermal energies before absorption (about 10^{-3} s). If the reproduction factor is k, and if there are N neutrons at time t, then there are, on the average, kN neutrons at time $t+\tau, k^{2}N$ at time $t+2\tau$, and so on. In a short time interval dt, the increase is

$$dN=(kN-N)\frac{dt}{\tau}$$

from which

$$N(t)=N_{0}e^{(k-1)t/\tau}$$

If $k=1$, then $N=$ constant; this would be the desired operating mode of a reactor. If $k<1$, the number of neutrons decays exponentially. If $k>1$, the number of neutrons grows exponentially with time, with a time constant char acterized by $\tau/(k-1)$. Even if the reactor is just slightly supercritical ($k=1.01$), the time constant is of order 0.1 s. It would be dangerous to operate a reactor in which the number of neutrons could increase by a factor of $e^{10}(=22{,}000)$ in 1 s . In practice, control of the number of neutrons is achieved by inserting into the pile a material such as cadmium, which is highly absorptive of thermal neutrons; the cadmium control rods are under mechanical control and can be gradually removed from the pile or rapidly inserted. If the pile is designed so that it is just slightly subcritical for prompt neutrons, the small number of delayed neutrons can be used to achieve critical operation, and since the delayed-neutron decay time constants are fairly long (seconds to minutes) the control rods can be manipulated to achieve a constant reaction rate.

UNIT IV: NUCLEAR DECAY

Beta decay – Continuous Beta spectrum – Fermi theory of beta decay - Comparative Half-life –Fermi Kurie Plot – mass of neutrino – allowed and forbidden decay — neutrino physics – Helicity - Parity violation - Gamma decay – multipole radiations – Angular Correlation – internal conversion – nuclear isomerism – angular momentum and parity selection rules.

Beta decay

The emission of ordinary negative electrons from the nucleus was among the earliest observed radioactive decay phenomena. The inverse process, capture by a nucleus of an electron from its atomic orbital, was not observed until 1938 when Alvarez detected the characteristic X rays emitted in the filling of the vacancy left by the captured electron. The Joliot-Curies in 1934 first observed the related process of positive electron (positron) emission in radioactive decay, only two years after the positron had been discovered in cosmic rays. These three nuclear processes are closely related and are grouped under the common name beta (β) decay.

The most basic β decay process is the conversion of a proton to a neutron or of a neutron into a proton. In a nucleus, β decay changes both Z and N by one unit: $Z \rightarrow Z \pm 1, N \rightarrow N \mp 1$ so that $A = Z + N$ remains constant. Thus β decay provides a convenient way for an unstable nucleus to "slide down" the mass parabola (Figure 3.18, for example) of constant A and to approach the stable isobar.

In contrast with α decay, progress in understanding β decay has been achieved at an extremely slow pace, and often the experimental results have created new puzzles that challenged existing theories. Just as Rutherford's early experiments showed α particles to be identical with ^{4}He nuclei, other early experiments showed the negative β particles to have the same electric charge and charge-to-mass ratio as ordinary electrons. In Section 1.2, we discussed the evidence against the presence of electrons as nuclear constituents, and so we must regard the β decay process as "creating" an electron from the available decay energy at the instant of decay; this electron is then immediately ejected from the nucleus. This situation contrasts with α decay, in which the α particle may be regarded as having a previous existence in the nucleus.

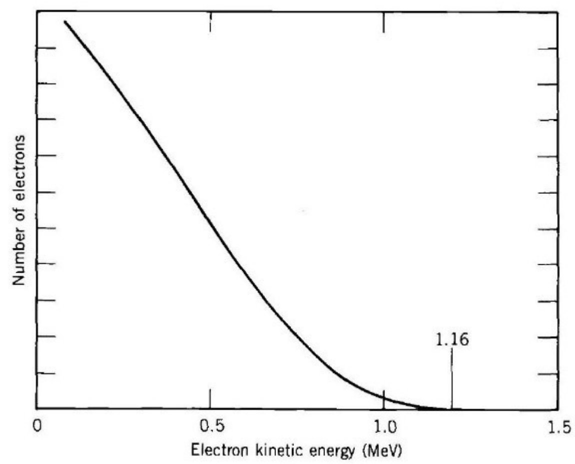
The basic decay processes are thus:

$\mathrm{n} \rightarrow \mathrm{p}+\mathrm{e}^{-}$ negative beta decay (β^{-})

$\mathrm{p} \rightarrow \mathrm{n}+\mathrm{e}^{+}$ positive beta decay (β^{+})

$\mathrm{p}+\mathrm{e}^{-} \rightarrow \mathrm{n}$ orbital electron capture (ε)

These processes are not complete, for there is yet another particle (a neutrino or antineutrino) involved in each. The latter two processes occur only for protons. The continuous electron distribution from the β decay of ^{210}Bi (also called RaE in the literature) bound in nuclei; they are energetically forbidden for free protons or for protons in hydrogen atoms.



Continues beta spectrum

The continuous energy distribution of the β-decay electrons was a confusing experimental result in the 1920s. Alpha particles are emitted with sharp, welldefined energies, equal to the difference in mass energy between the initial and final states (less the small recoil corrections); all α decays connecting the same initial and final states have exactly the same kinetic energies. Beta particles have a continuous distribution of energies, from zero up to an upper limit (the endpoint energy) which is equal to the energy difference between the initial and final states. If β decay were, like α decay, a two-body process, we would expect all of the β particles to have a unique energy, but virtually all of the emitted particles have a smaller energy. For instance, we might expect on the basis of nuclear mass differences that the β particles from ^{210}Bi would be emitted with a kinetic energy of 1.16 MeV , yet we find a continuous distribution from 0 up to 1.16 MeV (Figure 9.1).

An early attempt to account for this "missing" energy hypothesized that the β 's are actually emitted with 1.16 MeV of kinetic energy, but lose energy, such as by collisions with atomic electrons, before they reach the detection system. Such a possibility was eliminated by very precise calorimetric experiments that confined a β source and measured its decay energy by the heating effect. If a portion of the energy were transferred to the atomic electrons, a corresponding rise in temperature should be observed. These experiments showed that the shape of the spectrum shown in Figure 9.1 is a characteristic of the decay electrons themselves and not a result of any subsequent interactions.

To account for this energy release, Pauli in 1931 proposed that there was emitted in the decay process a second particle, later named by Fermi the neutrino. The neutrino carries the "missing" energy and, because it is highly penetrating radiation, it is not stopped within the calorimeter, thus accounting for the failure of those experimerts to record its energy. Conservation of electric charge requires the neutrino to be electrically neutral, and angular momentum conservation and statistical considerations in the decay process require the neutrino to have (like the electron) a spin of $\frac{1}{2}$. Experiment shows that there are in fact two different kinds of neutrinos emitted in β decay (and yet other varieties emitted in other decay processes; see Chapter 18). These are called the neutrino and the antineutrino and indicated by ν and $\bar{\nu}$. It is the antineutrino which is emitted in β^{-}decay and the neutrino which is emitted in β^{+}decay and electron capture. In discussing β decay, the term "neutrino" is often used to refer to both neutrinos and antineutrinos, although it is of course necessary to distinguish between them in writing decay processes; the same is true for "electron."

To demonstrate β-decay energetics we first consider the decay of the free neutron (which occurs with a half-life of about 10 min),

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^{-} + \bar{\nu}$$

As we did in the case of α decay, we define the Q value to be the difference between the initial and final nuclear mass energies.

$$Q = \left(m_{\mathrm{n}} - m_{\mathrm{p}} - m_{\mathrm{e}} - m_{\bar{\nu}}\right) c^{2}$$

and for decays of neutrons at rest,

$$Q = T_{\mathrm{p}} + T_{\mathrm{e}} + T_{\bar{\nu}}$$

For the moment we will ignore the proton recoil kinetic energy T_{p}, which amounts to only 0.3 keV. The antineutrino and electron will then share the decay energy, which accounts for the continuous electron spectrum. The maximumenergy electrons correspond to minimum-energy antineutrinos, and when the antineutrinos have vanishingly small energies, $Q \simeq (T_{\mathrm{e}})_{\max}$. The measured maximum energy of the electrons is $0.782 \pm 0.013\mathrm{MeV}$. Using the measured neutron, electron, and proton masses, we can compute the Q value:

$$\begin{aligned} Q &= m_{\mathrm{n}}c^2 - m_{\mathrm{p}}c^2 - m_{\mathrm{e}}c^2 - m_{\bar{\nu}}c^2 \\ &= 939.573\mathrm{MeV} - 938.280\mathrm{MeV} - 0.511\mathrm{MeV} - m_{\bar{\nu}}c^2 \\ &= 0.782\mathrm{MeV} - m_{\bar{\nu}}c^2 \end{aligned}$$

Thus to within the precision of the measured maximum energy (about 13 keV) we may regard the antineutrino as massless. Other experiments provide more stringent upper limits, as we discussed, and for the present discussion we take the masses of the neutrino and antineutrino to be identically zero.

Conservation of linear momentum can be used to identify β decay as a three-body process, but this requires measuring the momentum of the recoiling nucleus in coincidence with the momentum of the electron. These experiments are difficult, for the low-energy nucleus ($T \leqslant$ keV) is easily scattered, but they have been done in a few cases, from which it can be deduced that the vector sum of the linear momenta of the electron and the recoiling nucleus is consistent with an unobserved third particle carrying the "missing" energy and having a rest mass of zero or nearly zero. Whatever its mass might be, the existence of the additional particle is absolutely required by these experiments, for the momenta of the electron and nucleus certainly do not sum to zero, as they would in a two-body decay.

Because the neutrino is massless, it moves with the speed of light and its total relativistic energy E_ν is the same as its kinetic energy; we will use E_ν to represent neutrino energies. (A review of the concepts and formulas of relativistic kinematics may be found in Appendix A.) For the electron, we will use both its kinetic energy T_{e} and its total relativistic energy E_{e}, which are of course related by $E_{\mathrm{e}} = T_{\mathrm{e}} + m_{\mathrm{e}}c^2$. (Decay energies are typically of order MeV ; thus the nonrelativistic approximation $T \ll mc^2$ is certainly not valid for the decay electrons, and we must use relativistic kinematics.) The nuclear recoil is of very low energy and can be treated nonrelativistically.

Let's consider a typical negative β-decay process in a nucleus:

$${}_{Z}^{A}\mathrm{X}_{N} \rightarrow {}_{Z+1}^{A}\mathrm{X}_{N-1}' + \mathrm{e}^{-} + \bar{\nu}$$

where m_{N} indicates nuclear masses. To convert nuclear masses into the tabulated neutral atomic masses, which we denote as $m({}^{A}\mathrm{X})$, we use

$$m({}^{A}\mathrm{X})c^{2}=m_{\mathrm{N}}({}^{A}\mathrm{X})c^{2}+Zm_{\mathrm{e}}c^{2}-\sum_{i=1}^{Z}B_{i}$$

where B_{i} represents the binding energy of the i th electron. In terms of atomic masses,

$$Q_{\beta^{-}}=\{[m({}^{A}\mathrm{X})-Zm_{\mathrm{e}}]-[m({}^{A}\mathrm{X}')-(Z+1)m_{\mathrm{e}}]-m_{\mathrm{e}}\}c^{2}$$

Notice that the electron masses cancel in this case. Neglecting the differences in electron binding energy, we therefore find

$$Q_{\beta^{-}}=[m({}^{A}\mathrm{X})-m({}^{A}\mathrm{X}')]c^{2}$$

where the masses are neutral atomic masses. The Q value represents the energy shared by the electron and neutrino:

$$Q_{\beta^{-}}=T_{\mathrm{e}}+E_{\bar{\nu}}$$

and it follows that each has its maximum when the other approaches zero:

$$(T_{\mathrm{e}})_{\max}=(E_{\bar{\nu}})_{\max}=Q_{\beta^{-}}$$

In the case of the ${}^{210}\mathrm{Bi}\rightarrow{}^{210}\mathrm{Po}$ decay, the mass tables give

$$\begin{aligned}Q_{\beta^{-}}&=[m({}^{210}\mathrm{Bi})-m({}^{210}\mathrm{Po})]c^{2}\\&=(209.984095\mathrm{u}-209.982848\mathrm{u})(931.502\mathrm{MeV/u})\\&=1.161\mathrm{MeV}\end{aligned}$$

Figure showed $(T_{\mathrm{e}})_{\max}=1.16\mathrm{MeV}$, in agreement with the value expected from $Q_{\beta^{-}}$. Actually, this is really not an agreement between two independent values. The value of $Q_{\beta^{-}}$is used in this case to determine the mass of ${}^{210}\mathrm{Po}$, with the mass of ${}^{210}\mathrm{Bi}$ determined from that of ${}^{209}\mathrm{Bi}$ using neutron capture. Equation 9.6 is used with the measured $Q_{\beta^{-}}$to obtain $m({}^{A}\mathrm{X}')$.

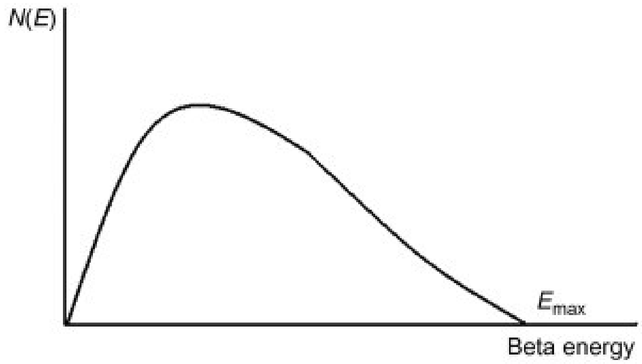
In the case of positron decay, a typical decay process is

$${}_{Z}^{A}\mathrm{X}_{N}\rightarrow{}_{Z-1}^{A}\mathrm{X}'_{N+1}+\mathrm{e}^{+}+\nu$$

and a calculation similar to the previous one shows

$$Q_{\beta^+} = [m(\,^A\mathrm{X}) - m(\,^A\mathrm{X}') - 2m_\mathrm{e}]c^2$$

again, using atomic masses. Notice that the electron masses do not cancel in this case.



For electron-capture processes, such as

$$^A_Z\mathrm{X}_N + \mathrm{e}^- \rightarrow \,^A_{Z-1}\mathrm{X}'_{N+1} + \nu$$

the calculation of the Q value must take into account that the atom X' is in an atomic excited state immediately after the capture. That is, if the capture takes place from an inner shell, the K shell for instance, an electronic vacancy in that shell results. The vacancy is quickly filled as electrons from higher shells make downward transitions and emit characteristic X rays. Whether one X ray is emitted or several, the total X-ray energy is equal to the binding energy of the captured electron. Thus the atomic mass of X' immediately after the decay is greater than the mass of X' in its atomic ground state by B_n, the binding energy of the captured n-shell electron ($n = \mathrm{K}, \mathrm{L}, \ldots$). The Q value is then

$$Q_\varepsilon = [m(\,^A\mathrm{X}) - m(\,^A\mathrm{X}')]c^2 - B_n$$

Positive beta decay and electron capture both lead from the initial nucleus $^A_Z\mathrm{X}_N$ to the final nucleus $^A_{Z-1}\mathrm{X}'_{N+1}$, but note that both may not always be energetically possible (Q must be positive for any decay process). Nuclei for which β^+decay is energetically possible may also undergo electron capture, but the reverse is not true-it is possible to have $Q > 0$ for electron capture while $Q < 0$ for β^+decay. The atomic mass energy difference must be at least $2m_\mathrm{e}c^2 = 1.022\mathrm{MeV}$ to permit β^+decay.

In positron decay, expressions of the form of Equations 9.7 and 9.8 show that there is a continuous distribution of neutrino energies up to Q_{β^+}(less the usually negligible nuclear recoil). In electron capture, however, the two-body final state results in unique values for the recoil energy and E_p. Neglecting the recoil, a monoenergetic neutrino with energy Q_ε is emitted.

All of the above expressions refer to decays between nuclear ground states. If the final nuclear state X' is an excited state, the Q value must be accordingly

Table Typical β-Decay Processes

Decay	Type	Q(MeV)	$t_{1/2}$
$^{23}Ne \rightarrow {}^{23}Na + e^- + \bar{\nu}$	β^-	4.38	38 s
$^{99}Tc \rightarrow {}^{99}Ru + e^- + \bar{\nu}$	β^-	0.29	2.1×10^5y
$^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
$^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
$^{15}O + e^- \rightarrow {}^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$	ε	0.43	1.0×10^5y

decreased by the excitation energy of the state:

$$Q_{\text{ex}} = Q_{\text{ground}} - E_{\text{ex}}$$

Table shows some typical β decay processes, their energy releases, and their half-lives.

Fermi theory of beta decay

In our calculation of α-decay half-lives in Chapter 8, we found that the barrier penetration probability was the critical factor in determining the half-life. In negative β decay there is no such barrier to penetrate and even in β^+decay, it is possible to show from even a rough calculation that the exponential factor in the barrier penetration probability is of order unity. There are other important differences between α and β decay which suggest to us that we must use a completely different approach for the calculation of transition probabilities in β decay:

(1) The electron and neutrino do not exist before the decay process, and therefore we must account for the formation of those particles. (2) The electron and neutrino must be treated relativistically. (3) The continuous distribution of electron energies must result from the calculation.

In 1934, Fermi developed a successful theory of β decay based on Pauli's neutrino hypothesis. The essential features of the decay can be derived from the basic expression for the transition probability caused by an interaction that is weak compared with the interaction that forms the quasi-stationary states. This is certainly true for β decay, in which the characteristic times (the half-lives, typically of order seconds or longer) are far longer than the characteristic nuclear time (10^{-20} s). The result of this calculation, treating the decay-causing interaction as a weak perturbation, is Fermi's Golden Rule, a general result for any transition rate previously given in Equation 2.79:

$$\lambda = \frac{2\pi}{\hbar}|V_{\mathrm{ff}}|^2\rho(E_{\mathrm{f}})$$

The matrix element V_{fi} is the integral of the interaction V between the initial and final quasi-stationary states of the system:

$$V_{\mathrm{fi}} = \int \psi_{\mathrm{f}}^* V \psi_{\mathrm{i}} dv$$

The factor $\rho(E_{\mathrm{f}})$ is the density of final states, which can also be written as dn/dE_{f}, the number dn of final states in the energy interval dE_{f}. A given transition is more likely to occur if there is a large number of accessible final states.

Fermi did not know the mathematical form of V for β decay that would have permitted calculations using above equations. Instead, he considered all possible forms consistent with special relativity, and he showed that V could be replaced with one of five mathematical operators O_X, where the subscript X gives the form of the operator O (that is, its transformation properties): $X =$ V (vector), A (axial vector), S (scalar), P (pseudoscalar), or T (tensor). Which of these is correct for β decay can be revealed only through experiments that study the symmetries and the spatial properties of the decay products, and it took 20 years (and several mistaken conclusions) for the correct V -A form to be deduced.

The final state wave function must include not only the nucleus but also the electron and neutrino. For electron capture or neutrino capture, the forms would be similar, but the appropriate wave function would appear in the initial state. For β decay, the interaction matrix element then has the form

$$V_{\mathrm{fi}} = g \int \left[\psi_{\mathrm{f}}^{*}\varphi_{\mathrm{e}}^{*}\varphi_{\nu}^{*}\right] O_X \psi_i \, dv$$

where now ψ_f refers only to the final nuclear wave function and φ_{e} and φ_ν give the wave functions of the electron and neutrino. The quantity in square brackets represents the entire final system after the decay. The value of the constant g determines the strength of the interaction; the electronic charge e plays a similar role in the interaction between an atom and the electromagnetic field.

The density of states factor determines (to lowest order) the shape of the beta energy spectrum. To find the density of states, we need to know the number of final states accessible to the decay products. Let us suppose in the decay that we have an electron (or positron) emitted with momentum $\boldsymbol{p}$ and a neutrino (or antineutrino) with momentum $\boldsymbol{q}$. We are interested at this point only in the shape of the energy spectrum, and thus the directions of $\boldsymbol{p}$ and $\boldsymbol{q}$ are of no interest. If we imagine a coordinate system whose axes are labeled p_x, p_y, p_z, then the locus of the points representing a specific value of $|\boldsymbol{p}| = \left(p_x^2 + p_y^2 + p_z^2\right)^{1/2}$ is a sphere of radius $p = |\boldsymbol{p}|$. More specifically, the locus of points representing momenta in the range dp at p is a spherical shell of radius p and thickness dp, thus having volume $4\pi p^2 dp$. If the electron is confined to a box of volume V (this step is taken only for completeness and to permit the wave function to be normalized; the actual volume will cancel from the final result), then the number of final electron states dn_{e}, corresponding to momenta in the range p to $p + dp$, is

$$dn_{\mathrm{e}} = \frac{4\pi p^2 dpV}{h^3}$$

where the factor h^3 is included to make the result a dimensionless pure number. * Similarly, the number of neutrino states is

$$dn_\nu = \frac{4\pi q^2 dqV}{h^3}$$

and the number of final states which have simultaneously an electron and a neutrino with the proper momenta is

$$d^2 n = dn_{\mathrm{e}} dn_\nu = \frac{(4\pi)^2 V^2 p^2 dp q^2 dq}{h^6}$$

The electron and neutrino wave functions have the usual free-particle form, normalized within the volume V :

$$\varphi_{\mathrm{c}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{p}\cdot\boldsymbol{r}/\hbar}$$
$$\varphi_\nu(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar}$$

For an electron with 1 MeV kinetic energy, $p = 1.4\mathrm{MeV}/c$ and $p/\hbar = 0.007\ \mathrm{fm}^{-1}$. Thus, over the nuclear volume, $pr \ll 1$ and we can expand the exponentials, keeping only the first term:

$$e^{i\boldsymbol{p}\cdot\boldsymbol{r}/\hbar} = 1 + \frac{i\boldsymbol{p}\cdot\boldsymbol{r}}{\hbar} + \cdots \cong 1$$
$$e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar} = 1 + \frac{i\boldsymbol{q}\cdot\boldsymbol{r}}{\hbar} + \cdots \cong 1$$

This approximation is known as the allowed approximation. In this approximation, the only factors that depend on the electron or neutrino energy come from the density of states. Let's assume we are trying to calculate the momentum and energy distributions of the emitted electrons. The partial decay rate for electrons and neutrinos with the proper momenta is

$$d\lambda = \frac{2\pi}{\hbar} g^2 |M_{\mathrm{fi}}|^2 (4\pi)^2 \frac{p^2 dp q^2}{h^6} \frac{dq}{dE_{\mathrm{f}}}$$

where $M_{\mathrm{fi}} = \int \psi_{\mathrm{f}}^* O_X \psi_{\mathrm{i}} dv$ is the nuclear matrix element. The final energy E_{f} is just $E_{\mathrm{e}} + E_\nu = E_{\mathrm{c}} + qc$, and so $dq/dE_{\mathrm{f}} = 1/c$ at fixed E_{c}. As far as the shape of the electron spectrum is concerned, all of the factors in Equation 9.20 that do not involve the momentum (including M_{fi}, which for the present we assume to be independent of p) can be combined into a constant C, and the resulting distribution gives the number of electrons with momentum between p and $p + dp$:

$$N(p)dp = Cp^2 q^2 dp$$

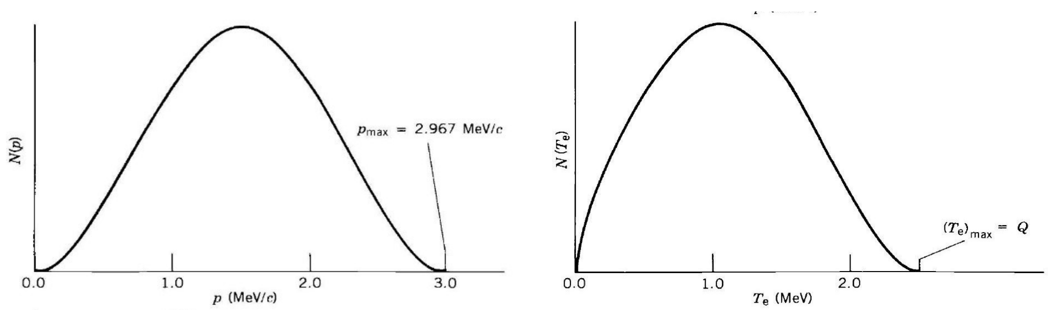
If Q is the decay energy, then ignoring the negligible nuclear recoil energy,

$$q = \frac{Q - T_{\mathrm{e}}}{c} = \frac{Q - \sqrt{p^2c^2 + m_{\mathrm{e}}^2c^4} + m_{\mathrm{e}}c^2}{c}$$

and the spectrum shape is given by

$$N(p) = \frac{C}{c^2}p^2(Q - T_{\mathrm{e}})^2$$
$$= \frac{C}{c^2}p^2\left[Q - \sqrt{p^2c^2 + m_{\mathrm{e}}^2c^4} + m_{\mathrm{e}}c^2\right]^2$$

This function vanishes at $p = 0$ and also at the endpoint where $T_{\mathrm{e}} = Q$; its shape is shown in Figure.



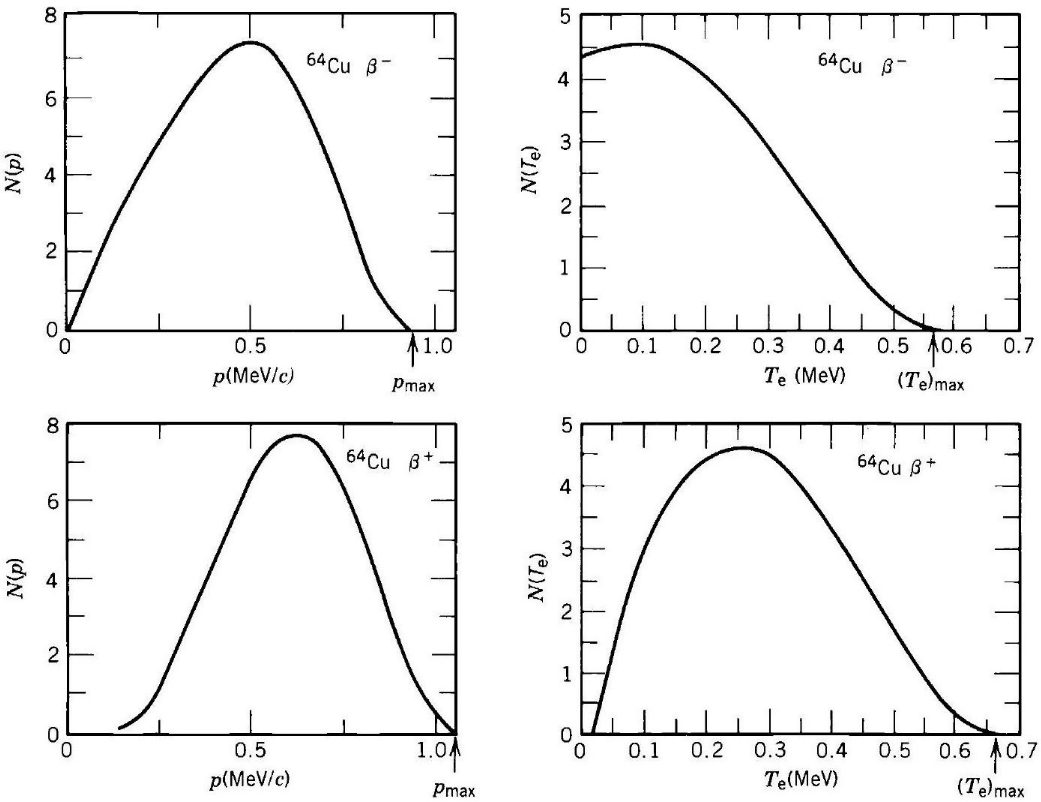
Expected electron energy and momentum distributions.

These distributions are drawn for $Q = 2.5$MeV.

More frequently we are interested in the energy spectrum, for electrons with kinetic energy between T_{e} and $T_{\mathrm{e}} + dT_{\mathrm{e}}$. With $c^2pdp = (T_{\mathrm{e}} + m_{\mathrm{e}}c^2)dT_{\mathrm{e}}$, we have

$$N(T_{\mathrm{e}}) = \frac{C}{c^5}(T_{\mathrm{e}}^2 + 2T_{\mathrm{e}}m_{\mathrm{e}}c^2)^{1/2}(Q - T_{\mathrm{e}})^2(T_{\mathrm{e}} + m_{\mathrm{e}}c^2)$$

This distribution, which also vanishes at $T_{\mathrm{e}} = 0$ and at $T_{\mathrm{e}} = Q$. The $\beta^+$and β^-decays of ^{64}Cu are compared with the predictions of the theory. There are systematic differences between theory and experiment. These differences originate with the Coulomb interaction between the β particle and the daughter nucleus. Coulomb repulsion of $\beta^+$by the nucleus, giving fewer low-energy positrons, and a Coulomb attraction of β^-, giving more low-energy electrons. From the more correct standpoint of quantum mechanics, we should instead refer to the change in the electron plane wave. The quantum mechanical calculation of the effect of the nuclear Coulomb field on the electron wave function is beyond the level of this text. It modifies the spectrum by introducing an additional factor, the Fermi function $F(Z', p)$ or $F(Z', T_e)$, where Z' is the atomic number of the daughter nucleus. Finally, we must



Momentum and kinetic energy spectra of electrons and positrons emitted in the decay of ^{64}Cu.

The differences arise from the Coulomb interactions with the daughter nucleus. From R. D. Evans, The Atomic Nucleus (New York: McGraw-Hill, 1955). consider the effect of the nuclear matrix element, M_{fi}, which we have up to now assumed not to influence the shape of the spectrum. This approximation (also called the allowed approximation) is often found to be a very good one, but there are some cases in which it is very bad-in fact, there are cases in which M_{fi} vanishes in the allowed approximation, giving no spectrum at all! In such cases, we must take the next terms of the plane wave expansion, which introduce yet another momentum dependence. Such cases are called, somewhat incorrectly, forbidden decays; these decays are not absolutely forbidden, but as we will learn subsequently, they are less likely to occur than allowed decays and therefore tend to have longer half-lives. The degree to which a transition is forbidden depends on how far we must take the expansion of the plane wave to find a nonvanishing nuclear matrix element. Thus, the first term beyond the 1 gives first-forbidden decays, the next term gives second-forbidden, and so on. We will see in Section 9.4 how the angular momentum and parity selection rules restrict the kinds of decay that can occur.

The complete β spectrum then includes three factors:

1. The statistical factor $p^2(Q - T_e)^2$, derived from the number of final states accessible to the emitted particles.
2. The Fermi function $F(Z', p)$, which accounts for the influence of the nuclear Coulomb field.
3. The nuclear matrix element $|M_{fi}|^2$, which accounts for the effects of particular initial and final nuclear states and which may include an additional electron and neutrino momentum dependence $S(p, q)$ from forbidden terms:

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q)$$

Comparative half – life

To find the total decay rate, we must integrate Equation 9.20 over all values of the electron momentum p, keeping the neutrino momentum at the value determined, which of course also depends on p. Thus, for allowed decays,

$$\lambda = \frac{g^2|M_{fi}|^2}{2\pi^3\hbar^7c^3}\int_0^{p_{max}} F(Z', p)p^2(Q - T_e)^2 dp$$

The integral will ultimately depend only on Z' and on the maximum electron total energy E_0 (since $cp_{max} = \sqrt{E_0^2 - m_e^2c^4}$), and we therefore represent it as

$$f(Z', E_0) = \frac{1}{(m_e c)^3(m_e c^2)^2}\int_0^{p_{max}} F(Z', p)p^2(E_0 - E_e)^2 dp$$

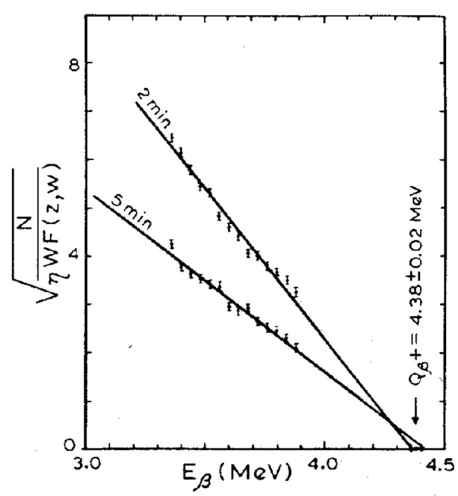
where the constants have been included to make f dimensionless. The function $f(Z', E_0)$ is known as the Fermi integral and has been tabulated for values of Z' and E_0.

Fermi-Kurie plot

In the allowed approximation, we can rewrite Equation 9.26 as

$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(Z', p)}}$$

and plotting $\sqrt{N(p)/p^2F(Z',p)}$ against T_e should give a straight line which intercepts the x axis at the decay energy Q. Such a plot is called a Kurie plot (sometimes a Fermi plot or a Fermi-Kurie plot). The linear nature of this plot gives us confidence in the theory as it has been developed, and also gives us a convenient way to determine the decay endpoint energy (and therefore the Q value).



Mass of neutrino

Neutrino mass is a fundamental topic in particle physics and cosmology. For a long time, neutrinos were assumed to be massless in the Standard Model of particle physics. However, experimental discoveries of neutrino oscillations (where neutrinos change their flavor as they travel) proved that neutrinos must have mass.

1. Experimental Evidence
   - Neutrino oscillation experiments (Super-Kamiokande, SNO, KamLAND, etc.) confirm that neutrinos have nonzero masses.
   - These experiments measure the differences in squared masses (Δm^2) between neutrino mass states rather than the absolute masses.
2. Mass Hierarchy
   - Normal Hierarchy: m1<m2<m3m_1 < m_2 < m_3m1<m2<m3 (like quarks and charged leptons)
   - Inverted Hierarchy: m3<m1<m2m_3 < m_1 < m_2m3<m1<m2

- The ordering is still unknown.

3. Upper Limits on Neutrino Mass

   - Cosmology provides a constraint: the sum of neutrino masses must be less than ~0.12 eV (from cosmic microwave background measurements).
   - The KATRIN experiment sets a direct upper limit on the electron neutrino mass: <0.8< 0.8<0.8 eV.

4. Theoretical Implications

   - The Standard Model does not predict neutrino mass; new physics like the seesaw mechanism is needed to explain why neutrinos are so light.
   - Neutrinos might be Majorana particles (their own antiparticles), which has implications for matter-antimatter asymmetry in the universe.

Allowed and forbidden decay

In the case of forbidden decays, the standard Kurie plot does not give a straight line, but we can restore the linearity of the plot if we instead graph $\sqrt{N(p)/p^2F(Z',p)S(p,q)}$ against T_e, where S is the momentum dependence that results from the higher-order term in the expansion of the plane wave. The function S is known as the shape factor; for certain first-forbidden decays, for example, it is simply p^2+q^2.

Neutrino physics

Neutrino physics is a rapidly evolving field that explores the properties and behavior of neutrinos, elusive elementary particles that interact only via the weak nuclear force and gravity. Neutrinos come in three flavors—electron, muon, and tau—and were initially thought to be massless under the Standard Model of particle physics. However, the discovery of neutrino oscillations, where neutrinos change flavors as they travel, provided strong evidence that they have mass. This groundbreaking discovery led to a Nobel Prize in 2015 and raised fundamental questions about the nature of neutrino mass. Unlike other particles, neutrinos do not have a fixed mass state; instead, their mass states mix with their flavor states, leading to oscillations. While experiments like Super-Kamiokande, SNO, and KamLAND have measured mass differences between neutrino states, the absolute mass scale remains unknown. The KATRIN experiment places an upper limit of 0.8 eV on the electron neutrino mass, and cosmological studies suggest the total neutrino mass must be below 0.12 eV. Theories like the seesaw mechanism attempt to explain why neutrino masses are so small, proposing the existence of much heavier right-handed neutrinos. Additionally, the question of whether neutrinos are Majorana particles (their own antiparticles) remains an open challenge with significant implications for particle physics and cosmology.

Helicity

Helicity is the projection of a particle's spin onto its momentum direction and plays a crucial role in beta decay. In the weak interaction, only left-handed particles (or right-handed antiparticles) participate, which has deep implications for neutrino physics.

In beta decay, a neutron (n) decays into a proton (p), emitting an electron (e^{-}) and an electron antineutrino $(\bar{\nu}_e)$:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Since neutrinos are massless (or nearly massless) and travel at speeds very close to the speed of light, their helicity is an essential quantum number. The electron antineutrino $(\bar{\nu}_e)$ is observed to be predominantly right-handed, while neutrinos in weak interactions are exclusively left-handed.

Helicity (h) is defined as:

$$h = \frac{\mathbf{S} \cdot \mathbf{p}}{|\mathbf{p}|}$$

where:

- S is the spin of the particle (for neutrinos, spin $S = \frac{1}{2}$),
- p is the momentum vector of the particle.

For a left-handed particle ($h = -1$), the spin is opposite to its momentum. For a right-handed particle ($h = +1$), the spin is aligned with its momentum.

Since neutrinos are always produced left-handed, this means that the weak interaction only allows one helicity state for the neutrino. Conversely, the antineutrino emitted in beta decay is always right-handed $(h = +1)$.

Parity violation

Weak interactions violate parity symmetry (P), meaning they distinguish between left-handed and right-handed particles. The famous Wu Experiment (1957) demonstrated that electrons emitted in polarized cobalt-60 beta decay tend to move opposite to the spin of the parent nucleus, proving that beta decay favors left-handed interactions.

The V-A (vector - axial vector) theory of weak interactions mathematically expresses this left-handed preference using the projection operator:

$$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$$

where γ^5 is a Dirac matrix. The weak interaction Lagrangian for beta decay is:

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \bar{\psi}_p \gamma^\mu (1 - g_A \gamma^5) \psi_n \cdot \bar{\psi}_e \gamma_\mu (1 - \gamma^5) \psi_{\nu_e}$$

where:

- G_F is the Fermi coupling constant,
- g_A is the axial vector coupling constant,
- γ^μ and γ^5 are Dirac matrices.

This equation shows that the weak interaction only couples to left-handed neutrinos and right-handed antineutrinos, confirming the helicity properties observed in beta decay.

Gamma decay

Most α and β decays, and in fact most nuclear reactions as well, leave the final nucleus in an excited state. These excited states decay rapidly to the ground state through the emission of one or more γ rays, which are photons of electromagnetic radiation like X rays or visible light. Gamma rays have energies typically in the range of 0.1 to 10 MeV , characteristic of the energy difference between nuclear states, and thus corresponding wavelengths between 10^4 and 100 fm . These wavelengths are far shorter than those of the other types of electromagnetic radiations that we normally encounter; visible light, for example, has wavelengths 10^6 times longer than γ rays.

The detail and richness of our knowledge of nuclear spectroscopy depends on what we know of the excited states, and so studies of γ-ray emission have become the standard technique of nuclear spectroscopy. Other factors that contribute to the popularity and utility of this method include the relative ease of observing γ rays (negligible absorption and scattering in air, for instance, contrary to the behavior of α and β radiations) and the accuracy with which their energies (and thus by deduction the energies of the excited states) can be measured. Furthermore, studying γ emission and its competing process, internal conversion, allows us to deduce the spins and parities of the excited states.

Multipole radiations

In deriving the dipole emission formula we only kept lowest order expansion:

$$\vec{A} \propto a_k^\dagger e^{i\vec{k}\cdot\vec{\epsilon}_{\vec{k}}} \approx a_k^\dagger(1 + i\vec{k}\cdot\vec{r} + \cdots)\vec{\epsilon}_k \to a_k^\dagger\vec{\epsilon}_k$$

This yields the typical dipole emission pattern:

$$W \propto \sin\theta^2 d\Omega$$

In QM the angular distribution is related to the photon angular momentum

Higher order terms in the expansion give rise to gamma emission with different angular-dependence pattern and higher angular momentum for the gamma photon emitted

$$\vec{A} \propto a_k^\dagger e^{i\vec{k}\cdot\vec{\epsilon}_\epsilon} \approx a_k^\dagger \sum_\ell \frac{(i\vec{k}\cdot\vec{r})^\ell}{\ell!}\vec{\epsilon}_k$$

Each ℓ term contributes to a different decay rate.

Electric multipole

$$\lambda(E\ell) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2}\frac{e^2}{\hbar c}\left(\frac{E}{\hbar c}\right)^{2\ell+1}\left(\frac{3}{\ell+3}\right)^2 c\langle|\hat{\vec{r}}|\rangle^{2\ell}$$
$$\langle\hat{r}\rangle\rangle \approx R_0 A^{1/3}$$

Rates:

$$\lambda(E1) = 1.0\times10^{14}A^{2/3}E^3$$
$$\lambda(E2) = 7.3\times10^{7}A^{4/3}E^5$$
$$\lambda(E3) = 34A^2E^7$$
$$\lambda(E4) = 1.1\times10^{-5}A^{8/3}E^9$$

Magnetic multipole

$$\lambda(M\ell)=\frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]}\frac{e^2}{\hbar c}\frac{E^{2\ell+1}}{\hbar c}\left(\frac{3}{\ell+3}\right)^2 c\langle|\hat{r}|\rangle^{2\ell-2}\left[\frac{\hbar}{m_p c}\left(\mu_p-\frac{1}{\ell+1}\right)\right]$$

Rates:

$$\lambda(M1)=5.6\times10^{13}E^3$$
$$\lambda(M2)=3.5\times10^{7}A^{2/3}E^5$$
$$\lambda(M3)=16A^{4/3}E^7$$
$$\lambda(M4)=4.5\times10^{-6}A^2E^9$$

Angular correlation

In gamma (γ) decay, an excited nucleus emits one or more gamma photons to transition to a lower energy state. The angular correlation of emitted gamma rays refers to the directional relationship between two successive gamma photons when a nucleus undergoes cascade transitions. These correlations provide critical information about nuclear spin, parity, and multipole transitions.

Consider a nucleus undergoing a cascade transition:

$$I_i \overset{\gamma_1}{>} I_m \overset{\gamma_2}{\underset{2}{}} > I_f$$

where:

- I_i is the initial nuclear spin,
- I_m is the intermediate spin,
- I_f is the final spin,
- γ_1 and γ_2 are the emitted photons.

The relative angular distribution of γ_2 with respect to γ_1 depends on:

- The multipolarity of the gamma transitions ($E1, M1, E2, M2$, etc.).
- The nuclear spin quantum numbers I_i, I_m, I_f.
- The mixing ratio (δ), which describes interference between multipole orders.

The angular correlation function between two gamma rays is given by:

$$W(\theta) = \sum_k A_k P_k(\cos\theta)$$

where:

- $W(\theta)$ is the probability of observing the second photon at an angle θ relative to the first photon.
- $P_k(\cos\theta)$ are Legendre polynomials.
- A_k are correlation coefficients that depend on nuclear spin values and multipole mixing.

For dipole ($\boldsymbol{E}1, M1$) and quadrupole ($\boldsymbol{E}2, M2$) transitions, common angular correlations are:

Transition Sequence	Expected Correlation $W(\theta)$
$E2 \to E2$	$1 + \frac{1}{2} P_2(\cos\theta) + \frac{3}{8} P_4(\cos\theta)$
$E2 \to M1$	$1 - \frac{1}{4} P_2(\cos\theta)$
$M1 \to E2$	$1 + \frac{1}{8} P_2(\cos\theta)$

These distributions are experimentally measured using gamma-gamma coincidence techniques.

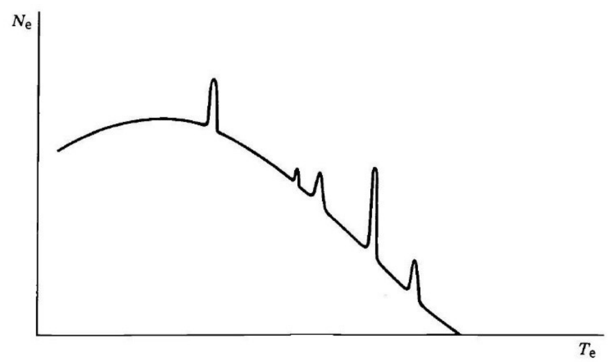
Internal conversion

Internal conversion is an electromagnetic process that competes with γ emission. In this case the electromagnetic multipole fields of the nucleus do not result in the emission of a photon; instead, the fields interact with the atomic electrons and cause one of the electrons to be emitted from the atom. In contrast to β decay, the electron is not created in the decay process but rather is a previously existing electron in an atomic orbit. For this reason internal conversion decay rates can be altered slightly by changing the chemical environment of the atom, thus changing somewhat the atomic orbits. Keep in mind, however, that this is not a two-step process in which a photon is first emitted by the nucleus and then knocks loose an orbiting electron by a process analogous to the photoelectric effect; such a process would have a negligibly small probability to occur.

The transition energy ΔE appears in this case as the kinetic energy T_e of the emitted electron, less the binding energy B that must be supplied to knock the electron loose from its atomic shell:

$$T_e = \Delta E - B$$

As we did in our discussion of nuclear binding energy, we take B to be a positive number. The energy of a bound state is of course negative, and we regard the binding energy as that which we must supply to go from that state up to zero energy. Because the electron binding energy varies with the atomic orbital.



A typical electron spectrum such as might be emitted from a radioactive nucleus. A few discrete conversion electron peaks ride on the continuous background from β decay. given transition ΔE there will be internal conversion electrons emitted with differing energies. The observed electron spectrum from a source with a single γ emission thus consists of a number of individual components; these are discrete components, however, and not at all continuous like the electrons emitted in β decay. Most radioactive sources will emit both β-decay and internal conversion electrons, and it is relatively easy to pick out the discrete conversion electron peaks riding on the continuous β spectrum .

The internal conversion process has a threshold energy equal to the electron binding energy in a particular shell; as a result, the conversion electrons are labeled according to the electronic shell from which they come: K, L, M, and so on, corresponding to principal atomic quantum numbers $n=1,2,3, \ldots$. Furthermore, if we observe at very high resolution, we can even see the substructure corresponding to the individual electrons in the shell. For example, the $\mathrm{L}(n=2)$ shell has the atomic orbitals $2\ \mathrm{s}_{1/2}, 2\mathrm{p}_{1/2}$, and $2\mathrm{p}_{3/2}$; electrons originating from these shells are called, respectively, $\mathrm{L}_{\mathrm{I}}, \mathrm{L}_{\mathrm{II}}$, and $\mathrm{L}_{\mathrm{III}}$ conversion electrons.

Following the conversion process, the atom is left with a vacancy in one of the electronic shells. This vacancy is filled very rapidly by electrons from higher shells, and thus we observe characteristic X-ray emission accompanying the conversion electrons. For this reason, when we study the γ emission from a radioactive source we usually find X rays near the low-energy end of the spectrum.

As an illustration of the calculation of electron energies, consider the β decay of ${}^{203}\mathrm{Hg}$ to ${}^{203}\mathrm{Tl}$, following which a single γ ray of energy 279.190 keV is emitted. To calculate the energies of the conversion electrons, we must look up the electron binding energies in the daughter Tl because it is from that atom that the electron emission takes place. (We will assume that the atomic shells have enough time to settle down between the β emission and the subsequent γ or conversion electron emission; this may not necessarily be true and will depend on the chemical environment and on the lifetime of the excited state.) Electron binding energies are conveniently tabulated in Appendix III of the Table of

Isotopes. For Tl , we find the following:

$$\begin{aligned}
B(\mathrm{K}) &= 85.529 \mathrm{keV} \\
B(\mathrm{L}_{\mathrm{I}}) &= 15.347 \mathrm{keV} \\
B(\mathrm{L}_{\mathrm{II}}) &= 14.698 \mathrm{keV} \\
B(\mathrm{L}_{\mathrm{III}}) &= 12.657 \mathrm{keV} \\
B(\mathrm{M}_{\mathrm{I}}) &= 3.704 \mathrm{keV}
\end{aligned}$$

and so on through the M, N, and O shells. We therefore expect to find conversion electrons emitted with the following energies:

$$\begin{aligned}
T_{\mathrm{e}}(\mathrm{K}) &= 279.190-85.529=193.661 \mathrm{keV} \\
T_{\mathrm{e}}(\mathrm{L}_{\mathrm{I}}) &= 279.190-15.347=263.843 \mathrm{keV} \\
T_{\mathrm{e}}(\mathrm{L}_{\mathrm{II}}) &= 279.190-14.698=264.492 \mathrm{keV} \\
T_{\mathrm{e}}(\mathrm{L}_{\mathrm{III}}) &= 279.190-12.657=266.533 \mathrm{keV} \\
T_{\mathrm{e}}(\mathrm{M}_{\mathrm{I}}) &= 279.190-3.704=275.486 \mathrm{keV}
\end{aligned}$$

The electron spectrum for ^{203}Hg. You can see the continuous β spectrum as well as the electron lines at the energies we have calculated.

One feature that is immediately apparent is the varying intensities of the conversion electrons from the decay. This variation, as we shall see, depends on the multipole character of the radiation field; in fact, measuring the relative probabilities of conversion electron emission is one of the primary ways to determine multipole characters.

In some cases, internal conversion is heavily favored over γ emission; in others it may be completely negligible compared with γ emission. As a general rule, it is necessary to correct for internal conversion when calculating the probability for γ emission. That is, if we know the half-life of a particular nuclear level, then the total decay probability λ_{t} (equal to $0.693/t_{1/2}$) has two components, one (λ_γ) arising from γ emission and another (λ_e) arising from internal conversion:

$$\lambda_t = \lambda_\gamma + \lambda_e$$

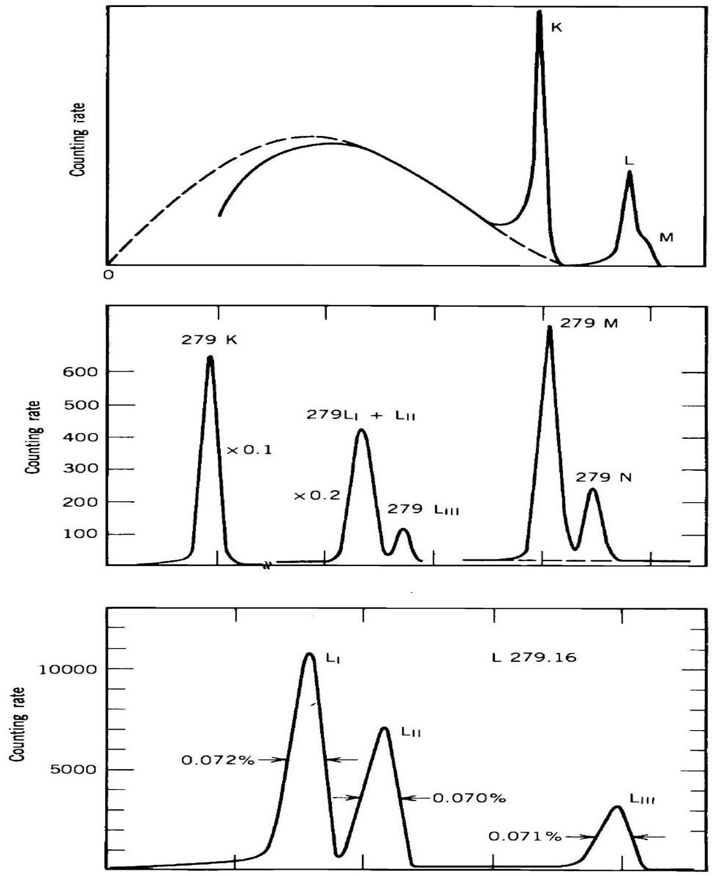
The level decays more rapidly through the combined process than it would if we considered γ emission alone. It is (as we shall see) convenient to define the internal conversion coefficient α as

$$\alpha = \frac{\lambda_{\mathrm{e}}}{\lambda_\gamma}$$

That is, α gives the probability of electron emission relative to γ emission and ranges from very small (≈ 0) to very large. The total decay probability then becomes

$$\lambda_t = \lambda_\gamma(1 + \alpha)$$

We let α represent the total internal conversion coefficient and define partial coefficients representing the individual atomic shells:



$$\lambda_t = \lambda_\gamma + \lambda_{e,K} + \lambda_{e,L} + \lambda_{e,M} + \cdots$$

Electron spectrum from the decay of ^{203}Hg. At top, the continuous β spectrum can be seen, along with the K and unresolved L and M conversion lines. In the middle is shown the conversion spectrum at higher resolution; the L and M lines are now well separated, and even L_{III} is resolved. At yet higher resolution (bottom) L_1 and L_{11} are clearly separated. Sources: (top) A. H. Wapstra et al., Physica 20, 169 (1954); (middle) Z. Sujkowski, Ark. Fys. 20, 243 (1961); (bottom) C. J. Herrlander and R. L. Graham, Nucl. Phys. 58, 544 (1964). and thus

$$\alpha = \alpha_K + \alpha_L + \alpha_M + \cdots$$

Of course, considering the subshells, we could break these down further, such as

$$\alpha_L = \alpha_{L_I} + \alpha_{L_{II}} + \alpha_{L_{III}}$$

and similarly for other shells. The calculation of internal conversion coefficients is a difficult process, beyond the level of the present text. Let's instead try to justify some of the general results and indicate the way in which the calculation differs from the similar calculation for γ emission. Because the process is electromagnetic in origin, the matrix element that governs the process is quite similar to that of with two exceptions: the initial state includes a bound electron, so that $\psi_i = \psi_{i,N}\psi_{i,e}$ where N indicates the nuclear wave function and e indicates the electron wave function. Similarly, $\psi_{\mathrm{f}} = \psi_{\mathrm{f,N}}\psi_{\mathrm{f,e}}$ where in this case $\psi_{\mathrm{f,e}}$ is the free-particle wave function $e^{-i\boldsymbol{k}\cdot r_e}$. To a very good approximation, the atomic wave function varies relatively little over the nucleus, and we can replace $\psi_{i,\mathrm{e}}(\boldsymbol{r}_{\mathrm{e}})$ with its value at $\boldsymbol{r}_{\mathrm{e}} = 0$. The important detail, however, is that all of the specifically nuclear information is contained in $\psi_{i,\mathrm{N}}$ and $\psi_{\mathrm{f,N}}$, and that the same electromagnetic multipole operator $m(\sigma L)$ governs both γ emission and internal conversion. The nuclear part of the matrix element of Equation 10.9 is therefore identical for both processes:

$$\lambda_\gamma(\sigma L) \propto |m_{\mathrm{fi}}(\sigma L)|^2$$

and thus the internal conversion coefficient α, the ratio of λ_{e} and λ_γ, is independent of the details of nuclear structure. The coefficient α will depend on the atomic number of the atom in which the process occurs, and on the energy of the transition and its multipolarity (hence, indirectly on the nuclear structure). We can therefore calculate and display general tables or graphs of α for different Z, T_{e}, and L.

We are oversimplifying here just a bit because the electron wave function $\psi_{\mathrm{i,e}}$ does penetrate the nucleus and does sample the specific nuclear wave function, but it has a very slight, usually negligible, effect on the conversion coefficient.

A nonrelativistic calculation gives the following instructive results for electric (E) and magnetic (M) multipoles:

$$\alpha(EL) \cong \frac{Z^3}{n^3}\left(\frac{L}{L+1}\right)\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_{\mathrm{e}}c^2}{E}\right)^{L+5/2}$$

$$\alpha(ML) \cong \frac{Z^3}{n^3}\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^4\left(\frac{2m_{\mathrm{e}}c^2}{E}\right)^{L+3/2}$$

In these expressions, Z is the atomic number of the atom in which the conversion takes place (the daughter, in the case of transitions following β decay) and n is the principal quantum number of the bound electron wave function; the factor $(Z/n)^3$ comes from the term $\left|\psi_{\mathrm{i,e}}(0)\right|^2$ that appears in the conversion rate.

Nuclear isomerism

The occurrence of long lived, low-lying excited states is rather common among nuclei of intermediate and large masses. Observed life times vary over wide limits, from 10^{-10}sec to 10^8 years. These delayed transitions are called isomeric transitions and the states from which they originate are called isomeric states or isomeric levels. Nuclear species which have the same atomic and mass numbers, but have different radioactive properties, are called nuclear isomers and their existence is referred to as nuclear isomerism. Nuclides that are isomeric states of a given isotope differ from each other in energy and in angular momentum.

The phenomenon of nuclear isomerism was discovered by O . Hahn, in 1921. He found that UX_2 ($^{23m}_{91}$ Pa) and UZ($^{234}_{91}$ Pa) both have the same atomic number and the same mass number but have different half lives and emit different radiations. Both grow out of UX_1($^{24}_{50}Th$) by $\beta^{-\mathrm{decay.}}$ UX_2 has a half life of 1.18 minutes and emits three groups of electrons with end point energies of 2.31MeV(90%),1.50MeV (9%) and 0.58MeV(1%). On the other hand, UZ has a half life of 6.7 hours and emits four groups of electrons with end point energies of 0.16MeV(28%),0.32MeV(32%),0.53MeV(27%) and 1.13 MeV (13%). About 0.15% of the UX_2 nuclei decay to ground state UZ. Both UX_2 and UZ then decay to U_{II} ($^{234}_{92}U$) by electron emission.

Angular momentum and parity selection rules

Conservation of total angular momentum require and parity dictates that:

$$\vec{I}_f = \vec{I}_i + \vec{l}$$
$$\pi_f = (-1)^l \pi_i \qquad (E- \text{ type })$$
$$\pi_f = (-1)^{l+1} \pi_i \qquad (M- \text{ type })$$

Recalling the rules of quantized angular momentum addition, $\left|I_f - l\right| \leq I_i + l$, or $\Delta I = \left|I_f - I_i\right| \leq l \leq I_f + I_i$. Note that the emission of an electromagnetic decay photon can not be associated with a $0 \rightarrow 0$ transition. (The can occur via internal conversion, discussed later.) The above parity selection rules can also be stated as follows:

$$\Delta\pi = \text{ no } \Rightarrow \text{ even } E/ \text{ odd } M$$
$$\Delta\pi = \text{ yes } \Rightarrow \text{ odd } E/ \text{ even } M$$

Some examples

$$\frac{3^{\pi}}{\underline{2}} \rightarrow \frac{5^{\pi}}{2}, \Delta\pi = \text{ no}$$

$\Delta I = 1$. Therefore, $M1, E2, M3, E4$ ...

$\frac{3^{\pi}}{\underline{2}} \rightarrow \frac{5\pi}{2}, \Delta\pi = \text{yes}$

$\Delta I = 1$. Therefore, $E1, M2, E3, M4$ ...

$\underline{0^{\pi} \rightarrow 0^{\pi}, \Delta\pi = \text{ unknown}}$

$\Delta I = 4$. Therefore, $E4$ if $\Delta\pi =$ yes, $M4$ otherwise.

$\underline{2}^{+} \rightarrow 0^{+}$

This is an $E2$ transition.

UNIT V: ELEMENTARY PARTICLES

Classification of Elementary Particles – Types of Interaction and conservation laws – Families of elementary particles – Isospin – Quantum Numbers – Strangeness – Hypercharge and Quarks –SU (2) and SU (3) groups-Gell Mann matrices– Gell Mann Okuba Mass formula-Quark Model. Standard model of particle physics – Higgs boson

Classification of Elementary Particles

Exploring the structure of atoms may give the impression that matter is composed solely of electrons, protons, and neutrons. However, research, especially on high-energy cosmic ray particles, has uncovered the existence of many additional nuclear particles. These subatomic or elementary particles are termed "elementary" because they have no internal structure. The classification of these particles is shown in the figure below, dividing them into two main groups: bosons and fermions. Bosons have intrinsic angular momentum that is an integer multiple of ħ, while fermions possess half-integer spin.

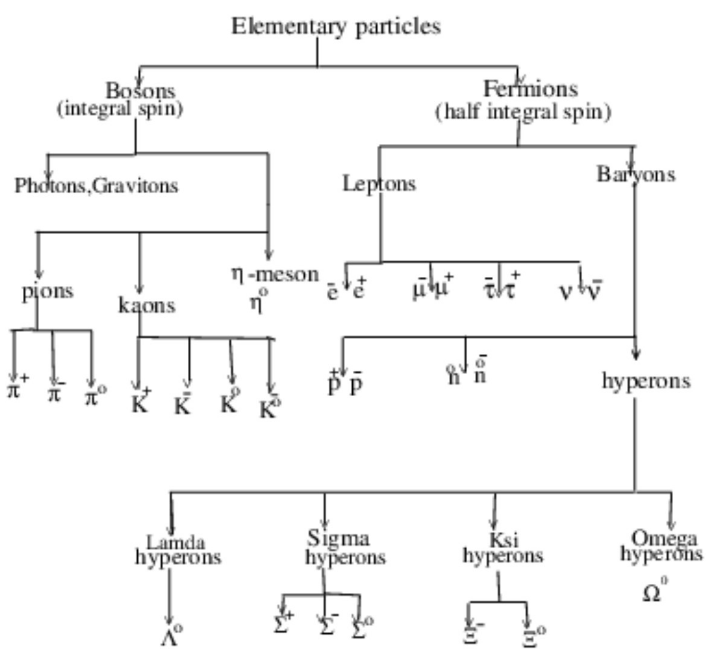


Figure: Classification of Elementary Particles

Types of Interaction

In particle physics, four fundamental interactions govern the behavior of elementary particles. These interactions determine how particles interact, decay, or transform. Along with these interactions, several conservation laws play a crucial role in maintaining the stability of physical processes.

The fundamental interactions may be defined as the fundamental forces that act between the elementary particles of which all matter is assumed to be composed. The fundamental interactions may be classified as (i) Strong interaction (ii) Electromagnetic interaction (iii) Weak interaction and (iv) Gravitational interaction. All known processes in nature (from sub nuclear to extra galactic ie. microscopic to macroscopic) can be considered as a manifestation of one or more of these interactions. The following table shows the summary of four interactions

Interaction	Relative Strength	Range	Mediating Particle	Acts On	Key Effects
Strong	1 (Strongest)	Short-range ($\sim 10^{-15}$ m)	Gluon (g)	Quarks & Hadrons (Baryons, Mesons)	Holds nuclei together, binds quarks in hadrons
Electromagnetic	10^{-21}	Infinite ($1/r^2$ decay)	Photon (γ)	Charged Particles	Governs atomic/molecular structure, light
Weak	10^{-13}	Very short ($\sim 10^{-18}$ m)	W^+, W^-, Z^0 bosons	All Fermions (Quarks & Leptons)	Causes beta decay, neutrino interactions
Gravitational	10^{-39}	Infinite ($1/r^2$ decay)	Graviton (hypothetical)	All particles with mass	Governs planetary motion, large-scale structure

Families of Elementary Particles

Baryons:

This group consists of protons and particles heavier than protons. Protons and neutrons, collectively known as nucleons, belong to this category, while the remaining particles are called hyperons. Each baryon has a corresponding antiparticle. If a baryon number of +1 is assigned to baryons and -1 to antibaryons, then in any closed system, interactions or decays preserve the total baryon number, following the law of baryon conservation. Hyperons form a special class of baryons, characterized by a decay time of approximately 10^{-10} seconds and a mass that lies between that of a neutron and a deuteron. Their decay time is significantly longer than their formation time, which is around 10^{-3} seconds.

Leptons:

This category includes electrons, photons, neutrinos, and muons.

Mesons:

These particles have a rest mass ranging between approximately 250 MeV and 1000 MeV. Mesons play a crucial role in mediating interactions between particles within the nucleus. Pions, kaons, and η-mesons collectively fall under this group. Together, baryons and mesons are classified as hadrons, which are the particles involved in strong interactions.

Particles and Anti-particles

Electron and Positron. The positron and the electron are said to be antiparticles. They have the same mass and the same spin but opposite charge. They annihilate each other with the emission of photons, when they come in contact with each other. Existence of an antiparticle for the electron was actually predicted by Dirac. Positron was discovered by Anderson in 1932. Proton and antiproton. The antiproton, was established in 1955 by Segre, Chamberlain, and their collaborators. Antiprotons were produced by bombarding protons in a target with 6 GeV protons, thereby inducing the reaction.

$$p + p \text{ (+energy)} \rightarrow p + p + p + p$$

Antiprotons interact strongly with matter and annihilate with protons.

$$P+ \bar{p} \rightarrow \pi^{+} + \pi^{-} + \pi^{+} + \pi^{-} + \pi^{0}$$

Class	*Name*	*Symbol*	*Spin*	*B*	*Le*	*Lμ*	*S*	*Y*	*I*
LEPTON	e- neutrino	ν_e	½	0	+1	0			
	μ – neutrino	ν_μ	½	0	0	+1			
	Electron	e^-	½	0	+1	0			
	Muon	μ^-	½	0	0	+1			
MESON	Pion	$\bar{\pi}, \pi^+$ π^0	0	0	0	0	0	0	1
	Kaon	K^+ K^0	0	0	0	0	+1	+1	1/2
	η meson	η^0	0	0	0	0	0	0	0
BARYON	Nucleon	*p,n*	½	+1	0	0	0	+1	1/2
	Λ hyperons	Λ^0	½	+1	0	0	-1	0	0
	Σ hyperons	Σ^+ Σ^0 Σ^-	½	+1	0	0	-1	0	1
	Ξ hyperons	Ξ^0 Ξ^-	½	+1	0	0	-2	-1	1/2
	Ω hyperons	Ω^-	3/2	+1	0	0	-3	-2	0

are restricted to $I, I-1, I-2 \ldots\ldots, 0, \ldots\ldots -(I-1), -I$. Hence I_3 is half-integral, if I is half-integral and integral or zero if I is integral

Neutron and antineutron.

The antiparticle of neutron, *antineutron*, discovered in 1956 by Cork, Lamberton and Wenzel.The nature of the antineutron is not very well known . Both neutron and antineutron have zero charge and the same mass. However, since neutron is supposed to have a certain internal charge distribution , it is expected that the antineutron has an internal charge distribution opposite to that of the neutron. Antineutron is quickly annihilated, either by a proton or a neutron, usually with the production of several pions. If an antineutron is not annihilated by a nucleon , it decays by the reaction.

$$\overline{p} + p \rightarrow \overline{n} + n$$

Neutrino and antineutrino:

The neutrino has a finite energy and momentum in flight. It travels with the speed of light *c*. It does not cause ionization on passing through matter. The antiparticle of neutrino is antineutrino. The distinction between the neutrino v and antineutrino $\bar{v}$ is a particularly interesting one. The spin of the neutrino is opposite in direction to the direction of its motion. The neutrino spins-counter clockwise. But the spin of the antineutrino is in the same direction as its direction of motion, it spins clockwise. Thus the neutrino moves through space in the manner of a left-handed screw, while the antineutrino does so in the manner of a right-handed screw. Thus neutrino possesses a " left handed" helicity; The antineutrino possesses a " right-handed " helicity, i.e., A neutrino and antineutrino differ only in the sense of their helicity. It is customary to The particle accompanying a positron a neutrino, v, while that accompanying an electron is called an antineutrino. Because of its lack of charge and magnetic moment, a neutrino has essentially no interaction with matter. This interaction is extremely weak.

$$\mathbf{n} \rightarrow \mathbf{p} + \mathbf{e}^{-} + \overline{\nu}$$

$$\mathbf{p} \rightarrow \mathbf{n} + \mathbf{e}^{+} + \mathrm{V}$$

Antimatter

In Atomic Physics, it has long been useful to consider an atom as composed of extra nuclear electrons and a nucleus. A positron and an anti proton could form an atom of anti hydrogen. Anti hydrogen would have a spectrum similar to that of ordinary hydrogen. Indeed, from a collection of anti protons, anti neutrons, and positrons, everything were made of antiparticles. Particle-antiparticle annihilation would occur with a tremendous release of energy.

Conservation laws of Elementary Particles

In classifying the various elementary particles, several discrete quantum numbers are used. We are already familiar with two such quantum numbers, namely those that describe a particles charge and spin. These quantum numbers, specify measurable physical properties and are always conserved. We know that all elementary charges are 0 or 1. Particles with integer spin obey the Bose-Einstein statistics and are called bosons. Particles with half odd integer spins obey the Fermi-Dirac statistics and are called fermions.

1. **Baryon number**. Each **baryon** is given a baryon number B=1,each corresponding anti baryon is given a baryon number B= - 1. All other particles have B = 0. The law of conservation of baryons states that the sum of the baryon number of all the particles after a reaction or decay must be the same as before. This rule ensures that a proton cannot change into an electron , even though a neutron can change into a proton. Baryon conservation ensures the stability of the proton against decaying into a particle of smaller mass. All normal baryons such as p^+, n^0, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω have the baryon of +1. All the corresponding anti particles known as anti baryons have the baryon number -1.
All the mesons have baryon number 0.

$\Lambda^0 \rightarrow p^+ + \pi^-$ ie. [1= 1+ 0]

Another example for the conservation of baryon number is

$n \rightarrow p^+ + e^- + \bar{\nu}$ [1 = 1 +0+ 0]

Hence the baryon number is conserved

2. Lepton number. The **Leptons** are supposed to possess a property called *Lepton number* (L). Since the neutrinos associated with electrons and with muons are recognized as different, we introduce two lepton numbers Le and L μ both of which must be conserved separately in particle reaction and decays. The number Le = 1 is assigned to the electron and the e-neutrino and Le = - 1 to their antiparticles. All other particles have Le = 0. Also the number L μ = 1 is assigned to the **muon** and the **μ – neutrino** and Lμ = - 1 to their antiparticles.

$\pi^- + p^+ \rightarrow \Lambda^0 + K^0$ [0+0 =0 +0]

3. **Strangeness number**. They were produced by high energy reactions but always in pairs i.e., if one particle of some kind is produced then simultaneously another different particle is also emitted. Associated production of **kaons** and **hyperons** which are known as strange particles. Decay through strong interactions in a very short time but this is not observed. Instead they decay slowly. Because of this strange behavior they were called as strange particles.

$$K^0 \rightarrow \pi^+ + \pi^- \qquad [+1 = 0+0]$$

Here the LHS is 1 and RHS is 0. So the reaction is not conserved for strangeness. It is found that S is conserved in all processes mediated by the strong and electromagnetic interactions. The multiple creation of particle with S not equal to 0 is the result of the conservation principle. An example is the proton-proton collision:

$$P^+ + P^+ \rightarrow \Lambda^0 + K^0 + P^+ + \pi^+$$

strangeness number S ; 0 + 0 → (-1) + 1 + 0 + 0

On the other hand, S can change in an event governed by the weak interaction. The decays of kaons and hyperons proceed via the weak interaction and accordingly are extremely slow. Even the weak interaction, however, is unable to permit S to change by more than ±1 in a decay.

$$\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu_e \qquad [-1 = -1 + 0+0] \; ; \quad \Delta S=0$$

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \qquad [-1 = 0+0+0] \; ; \quad \Delta S = -1$$

$$\pi^- + p \rightarrow \Lambda^0 + K^0 \qquad [0+0 = -1 +1] \; ; \quad \Delta S=0$$

4. Isospin and Isospin quantum number.

As far as strong interactions are concerned, the neutron and the proton are two states of equal mass of a nucleon doublet. It is found that particles occur in multiplets. For example, singlets η^0 (eta-meson), Ω^-(omega hyperons) , Λ^0(lamda hyperon). Doublet : *p, n*, triplet, π^+ ,π^- ,π^0 (pions or Pi-mesons) . It is natural to think of the members of a multiplet as representing different charge states of a single fundamental entity. It has proved useful to categorize each multiplet according to the number of charge states it exhibits by a number I such that the multiplicity of the state is given by 2 *I*+ 1. Thus the nucleon multiplet is assigned *I* = ½, and its 2 x ½ + 1 = 2 states are the neutron and the proton. The pion multiplet has I = 1, and its 2 x 1 + 1 = 3 states are π^+ ,π^- ,π^0.

Isospin can be represented by a vector *I* in "isospin space" whose component in any specified direction is governed by a quantum number customarily denoted I_3. The possible values of I_3 are restricted to I, $I - 1$, $I - 2$…….., 0, ….. – ($I - 1$), - I. Hence I_3 is half-integral, if I is half integral and integral or zero if I is integral For the nucleon, $I = ½$ which means that I_3 can be either +1/2 or -1/2 ; The former is taken to represent the proton and the later neutron.

Hypercharge.

A quantity called **hypercharge (*Y*)** is conserved in strong interaction. Hypercharge is equal to the sum of the strangeness and baryon numbers of the particle , $\boldsymbol{Y = S + B}$. For mesons B=0, so the hypercharge equals the strangeness.

SU (2) and SU (3) groups

It has been found that, in general, the conservation law represents an invariance which corresponds in an appropriate symmetry operation. The set of operators that represents the symmetry constitutes the group from which the theory gets its name. The irreducible representations of a group consist of a number of states, quantities or objects to which the symmetry operations are applicable. Thus the appropriate group operation can transform any one of these states into another in the same representation. The fundamental representation is the one containing the smallest number of states for the particular group. Linear momentum is conserved if the system is in variant with respect to displacement in space. Angular momentum is conserved if invariance is with respect in angular displacement and energy is conserved if it is with respect to time. The simplest unitary group U (1) contains transformations which add a phase factor only to particle wave functions. The invariance under such transformations gives conservations of charge Q, baryon number B, lepton number L and hypercharge Y

SU(2) Symmetry

We know that proton and neutron are identical as far as the nuclear force is concerned, but differ in their electromagnetic interactions. Thus, it is possible to imagine a group of symmetry operators which could transform a neutron into a proton (or proton into a neutron) in the absence of an electromagnetic field. The proton and neutron would then form the fundamental representations of the group. The existence of such symmetry implies that something remains constant under the strong interaction. This is known as isospin and is ½ for proton as well as for neutron. The component of the isospin, $T3$ is + ½ for the proton and - ½ for the neutron. The operators of the symmetry group thus change the co-ordinates of isospin in such a way as to reverse the sign of *T3*. It can also be expressed as: the strong interactions are assumed to be invariant under rotations in the isotopic spin space. The particular symmetry group applicable to isospin conservation is a form of unitary symmetry known as U(2), which can be expressed by a set of 2 x 2 matrices. This group may be reduced to a special unitary group SU (2), which is also written as SU2. It is special because a restriction reduces by unity the number of operators in the group. The two dimensions refer to the two basic states which make up the fundamental representation in this case. The restriction of reducing the number of operators 2 x 2 = 4 to three. The group is then said to have three generators.

By the use of the algebra of the *SU(2)* group it can be shown that all irreducible representations of the symmetry group consist of a multiplet of 2T + 1 states. All the members of the multiplet have the same isospin T and are essentially identical except for charge. If the symmetry was exact, i.e. isospin is strictly conserved, the components of a multiplet would differ in charge and *T3* . The *SU(2)* symmetry is violated by the electromagnetic interaction for which conservation of isospin is not applicable

Eightfold Way (SU (3) Symmetry)

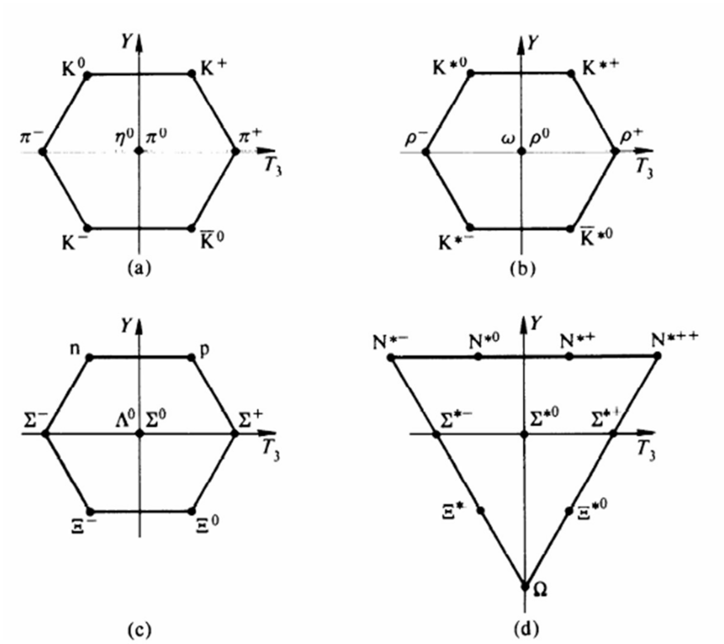
When and K particles were discovered, they were produced in pair (associated production) with longer life time. It was postulated that these new particles possessed a new additive quantum number, called strangeness S , conserved by strong interaction but violated in decays,

$$S(\Lambda^0) = -1, \quad S(K^0) = 1 \quad \cdots$$

Extension to other hadrons systematically, we can get a general relation,

$$Q = T_3 + \frac{Y}{2}$$

where Y = B+S is called hyperchargee; and B is the baryon number. This is known as Gell-Mann-Nishijima relation. Eight-fold way : Gell-Mann, Neeman When we group mesons or baryons with same spin and parity, we see that



Gell Mann matrices

The Gell-Mann matrices are the generators of the special unitary group SU(3), analogous to the Pauli matrices for SU(2). They are commonly used in quantum chromodynamics (QCD) to describe the behavior of quarks and gluons.

Definition

There are **eight Gell-Mann matrices**, denoted as λ_i ($i = 1, \ldots, 8$). These are **Hermitian, traceless,** and satisfy the **Lie algebra** of $SU(3)$:

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k$$

where f_{ijk} are the **structure constants** of $SU(3)$.

Properties

1. **Hermitian:** $\lambda_i^\dagger = \lambda_i$.
2. **Traceless:** $\mathrm{Tr}(\lambda_i) = 0$.
3. **Orthonormality:**

$$\mathrm{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}$$

4. **Commutation relations:**

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k$$

where f_{ijk} are the structure constants of $SU(3)$.

5. **Anticommutation relations:**

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} I + 2 d_{ijk} \lambda_k$$

where d_{ijk} are the **totally symmetric structure constants.**

Explicit Form The Gell-Mann matrices in the standard basis are:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$
$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$
$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Gell Mann Okuba Mass formula

Since SU(3) is not an exact symmetry, we want to see whether we can understand the pattern of the SU(3) breaking. Experimentally, SU(2) seems to be a good symmetry, we will assume isospin symmetry to set mu =md: We will assume that we can write the hadron masses as linear combinations of quark masses.

(a) o^- mesons

Here we assume that the meson masses are linear functions of quark masses, Where λ and m_0 are some constants with mass dimension. Eliminate the quark masses we get

$$4m_k^2 = m_\pi^2 + 3m_\eta^2$$

This known as the Gell-Mann Okubo mass formula. Experimentally, we hav $LHS = 4m_k^2 \simeq 0.98(Gev)^2$ while $RHS = m_\pi^2 + 3m^2 \simeq 0.92(Gev)^2$ This seems to show that this formula works quite well.

(b) $\frac{1}{2}^+$ baryon

The masses of $\frac{1}{2}^+$ baryons can be written as,

$$\begin{aligned} m_N &= m_0 + 3m_u \\ m_\Sigma &= m_o + 2m_u + m_s \\ m_\Xi &= m_o + m_u + 2m_s \\ m_\Lambda &= m_o + 2m_u + m_s \end{aligned}$$

The Gell-Mann-Okubo mass formula takes the form,

$$\frac{m_\Sigma + 3m_\Lambda}{2} = m_N + m_\Xi$$

Expermentally, $\frac{m_\Sigma + 3m_\Lambda}{2} \simeq 2.23\ Gev$, and $m_N + m_\Xi \simeq 2.25\ Gev$.

The Quark Model

Murray Gell-Mann and G. Zweig proposed the quark model in 1964. This theory is based on the idea that the hadrons are built up from a limited number of "fundamental" units, which have acquired the name quarks.

Quark	Symbol	Charge	Spin	Baryon Number (B)	Strangeness (S)	Isospin (I)	Isospin Projection (I_3)
Up	u	+2/3	½	1/3	0	½	+1/2
Down	d	-1/3	½	1/3	0	½	-1/2
Strange	s	-1/3	½	1/3	-1	0	0

u quark has electric charge $+\frac{2}{3}e$ and strangeness 0.

d quark has electric charge $-\frac{1}{3}e$ and strangeness 0.

s quark has electric charge $-\frac{1}{3}e$ and strangeness – 1.

Each quark has a baryon number of B = 1/3.

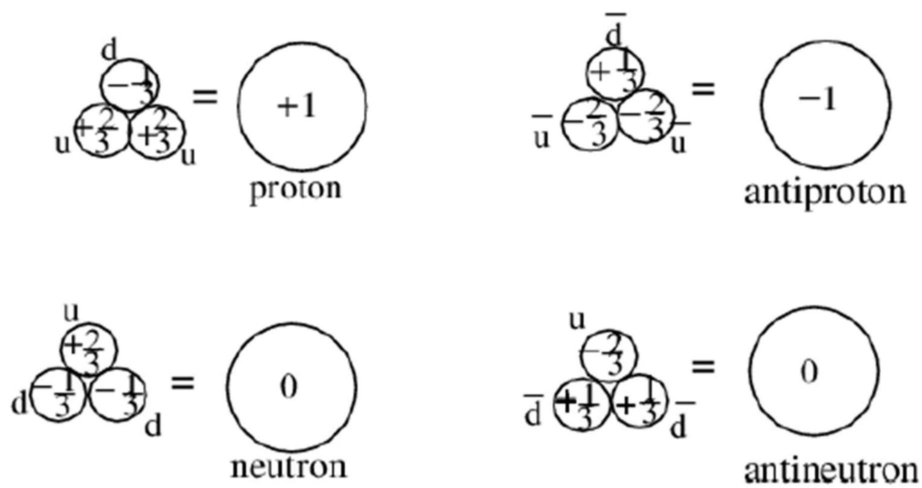


Figure: Constituents of proton and neutron in terms of quark

Each quark has an antiquark associated with it ($\bar{u}, \bar{d}, \bar{s}$. The magnitude of each of the quantum numbers for the antiquarks has the same magnitude as those for the quarks, but the sign is changed.

Compositions of hadrons according to the quark model

Hadrons may be baryons or mesons. A baryon is made up of three quarks. For example, the proton is made up of two *u* quarks and one *d* quarks *(uud).* For these quarks, the electric charges are + 2/3, + 2/3, and – 1/3, for a total value of + 1. The baryon number are + 1/3, +1/3 and + 1/3, for a total of + 1. The strangeness numbers are 0, 0 and 0 for a total strangeness of 0. All are in agreement with the quantum numbers for the proton. Quark models of the proton, antiproton, neutron and antineutron. Electric charges are given in units of *e.*

A meson is made up of one quark and one antiquark. For example, the π^+ meson is the combination of one quark either *u* or *d* and one antiquark either $\bar{u}, \bar{d}$. The Electric charges of these quarks are + 2/3 and + 1/3 for a total of +1, for a total of baryon number of 0. The strangeness numbers are 0 and 0 for a total of 0. All of these are in agreement with the quantum numbers for the pi-meson. Quarks all have spins of ½, which accounts for the observed half-integral spins of baryons and the 0 or 1 spins of mesons. All known hadrons can be explained in terms of the various quarks and their antiquarks. Quark contents of five hadrons and how they account for the observed charges, spins, and strangeness numbers of these particles is given in the table below.

Hadron	Quark Content	Baryon Number (B)	Charge (e)	Spin	Strangeness (S)
π^+ (Pi-plus)	u $\bar{d}$	0	+1	0	0
K^+ (Kaon-plus)	u $\bar{s}$	0	+1	0	+1
p^+ (Proton)	u u d	+1	+1	1/2	0
n^0 (Neutron)	d d u	+1	0	1/2	0
Ω^- (Omega-minus)	sss	+1	-1	3/2	-3

Coloured quarks and gluons: There were problems with the quark model, one of them being Ω^- hyperon. It was believed to contain three identical *s* quarks (*sss*). This violates the Pauli exclusion principle, that prohibits two or more fermions from occupying identical quantum states. The proton, neutron, and others with two identical quarks would violate this principle also. We can resolve this difficulty by assigning a new property to the quarks. We can regard this new property as an additional quantum number that can be used to label the three otherwise identical quarks in the Ω -. If this additional quantum number can take any one of three possible values, we can restore the Pauli‟s principle by giving each quark has a different value of this new quantum number, which is known as colour. The three colours are labeled red (*R*), blue (*B*), and green (*G*). The Ω – for example, would then SR , SB ,SG. The antiquark colours are antired (*R*) antiblue (*B*) and antigreen (*G*).

An essential component of the quark model with colour is that all observed meson and baryon states are "colourless", i.e., either colour, anticolour combinations in the case of mesons, or equal mixtures of R, B and G in the case of baryons. Since hadrons seem to be composed of quarks, the strong interaction between hadrons should ultimately be traceable to an interaction between quarks. The force between quarks can be modeled as an exchange force, mediated by the exchange of mass less spin –1 particles called gluons. Eight gluons have been postulated. The field that binds the quarks is a colour field. Colour is to the strong interaction between quarks as electric charge is to the electromagnetic interaction between electrons. It is the fundamental strong "charge" and is carried by the gluons. The gluons must therefore be represented as combinations of a colour and a possibly different anticolour. The gluons are massless and carry their colour and anticolour properties just as other particles may carry electric charge. For example, a gluon RB being exchanged by red and blue quarks. In effect the red quark emits its redness into a gluon and acquires blueness by also emitting antiblueness. The blue quark, on the other hand, absorbs the *RB* gluon, canceling its blueness and acquiring a red colour in the process.

Charm, Bottom, and Top. The charmed quark was suggested to explain the suppression of certain decay processes that are not observed. With only three quarks, the processes would proceed at measurable rates and should have been observed. The charm quark has a charge of 2/3 *e,* strangeness 0 and a charm quantum number of + 1. Other quarks have 0 charm.

Generation	Quark	Symbol	Charge (e)	Strangeness (S)	Charm (C)
1st	Up	**u**	+2/3	0	0
	Down	**d**	-1/3	0	0
2nd	Charm	**c**	+2/3	0	+1
	Strange	**s**	-1/3	-1	0
3rd	Top	**t**	+2/3	0	0
	Bottom	**b**	-1/3	0	0

Quantum Numbers – The quarks have quantum numbers. The *s*-quark has a quantum number called strangeness. The *C, B* and *T* quantum numbers are conserved in the strong and electromagnetic interactions and change by one unit in the weak interactions. This means that the number of quarks minus antiquarks for each *s, c, b* and *t* must remain constant in strong and electromagnetic interactions, whereas in the weak interaction there is a change of quark flavor with the preferred sequence $t \rightarrow b \rightarrow c \rightarrow s$. Since three of the quarks are needed to make a baryon, therefore, the baryon number is 1/3 for all the quarks. The quarks quantum numbers are summarized in table .

Table: Quark quantum numbers and properties

Quantum Number	*u*	*d*	*s*	*c*	*b*	*t*
Charge	2/3	-1/3	-1/3	2/3	-1/3	2/3
Mass (GeV/c^2)	0.39	0.39	0.51	1.55	~ 5	~ 30
Spin in *h*	1/2	1/2	1/2	1/2	1/2	1/2
Isospin *I*	1/2	1/2	0	0	0	0
Isospin Component I_3	1/2	-1/2	0	0	0	0
Baryon number *B*	1/3	1/3	1/3	1/3	1/3	1/3
Strangeness *S*	0	0	-1	0	0	0
Charm *C*	0	0	0	1	0	0
Botton *B*	0	0	0	0	-1	0
Top *T*	0	0	0	0	0	1

The isospin quantum number *T* is ½ and therefore $T3$=1/2 and -1/2 for the up and down quarks respectively. The quantum number *S* of strange quark and of beauty quark is -1. It is 1 for the charm and top quarks. The hypercharge is a quantum number related to quark strangeness and baryon number, whereas the isospin is a quantum number related to the *u*-*d* quark difference. The colour quantum number breaks the degeneracy and allows up to three quarks of the same flavor to occupy a single quantum state.

Quark Masses - Among the six quarks, the least massive members are the *u* and *d* quarks, each of same mass, around 0.39 GeV/c^2. The lighest baryons, nucleons, *Δ* particles, and the lightest mesons, pions must therefore be exclusively made of these two quarks. The s quark is more massive, around 0.51 GeV/ c^2. It carries a quantum number called strangeness and a necessary constituent of particles called strange particles (with non-zero strangeness), such as K-mesons, and baryon Λ. The *c*-quark is even more massive, having rest mass around 1.65 GeV/c^2. The *b*-quark has a rest mass around 5 GeV/c^2.

Mesons and Baryons – All hadrons are made of six quarks and their antiquarks. The properties of the quarks is inferred from the properties of mesons and baryons. To know the masses of the quarks from the known hadron masses, we need to know the strength of the interaction between quarks in the hadron. The hadrons are subdivided into two classes, baryons and mesons. Baryons are Fermions, this implies that quarks are also the fermions. Since the quark cannot exist as a free particle, the lightest fermion in the hadron family must therefore be made of three quarks. Thus

$$|p\rangle = |uud\rangle \quad \text{and} \quad |n\rangle = |u\,dd\rangle.$$

$$Q_p = \frac{2}{3} + \frac{2}{3} + \left(\frac{-1}{3}\right) = 1; \qquad Qn = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0\,.$$

Standard Model of Particle Physics

The Standard Model (SM) is a theory describing fundamental particles and their interactions (except gravity). It is based on quantum field theory (QFT) and the concept of gauge symmetries.

Fundamental Particles in the Standard Model

The Standard Model consists of fermions (matter particles) and bosons (force carriers).

2.1. Fermions (Matter Particles)

- Spin-1/2 particles classified into quarks and leptons.
- They obey the Pauli exclusion principle.

Type	Name	Symbol	Charge	Mass (approx.)
Quarks	Up	u	$+\frac{2}{3}e$	2.3 MeV
	Down	d	$-\frac{1}{3}e$	4.8 MeV
	Charm	c	$+\frac{2}{3}e$	1.27 GeV

Type	Name	Symbol	Charge	Mass (approx.)
	Strange	s	$-\frac{1}{3}e$	96 MeV
	Top	t	$+\frac{2}{3}e$	173 GeV
	Bottom	b	$+\frac{1}{3}e$	4.18 GeV
Leptons	Electron	e^-	−e	0.511 MeV
	Electron Neutrino	ν_e	0	< 1 eV
	Muon	μ	−e	105.7 MeV
	Muon Neutrino	ν_μ	0	< 1 eV
	Tau	τ−	−e	1.78 GeV
	Tau Neutrino	ν_τ	0	< 1 eV

2.2. Bosons (Force Carriers)

- **Gauge bosons** mediate the fundamental forces.

Force	Carrier	Symbol	Charge	Mass
Electromagnetism	Photon	γ	0	0
Weak Force	W boson	$W^\pm$	±e	80.4 GeV
	Z boson	Z^0	0	91.2 GeV
Strong Force	Gluon	g	0	0
Higgs Mechanism	Higgs Boson	H	0	125 GeV

4. Fundamental Forces in the Standard Model

Force	**Particles Affected**	**Carrier**	**Strength (Relative to Strong)**	**Range**
Strong	Quarks, Gluons	Gluon g	1	Short ($\sim10^{-15}$ m)
Electromagnetic	Charged Particles	Photon γ	$\sim10^{-2}$	Infinite
Weak	All Fermions	$W^{\pm}, Z^{0}$	$\sim10^{-5}$	Short ($\sim10^{-18}$ m)

Higgs Boson

The Higgs boson is a fundamental particle in the Standard Model (SM) of particle physics**.** It was predicted in 1964 by Peter Higgs and others and discovered in 2012 at the Large Hadron Collider (LHC)**.** The Higgs boson plays a crucial role in the Higgs mechanism, which gives mass to fundamental particles

Higgs Mechanism and Mass Generation

- The Higgs field is a quantum field that permeates all space.
- Fundamental particles interact with this field to acquire mass.
- The Higgs mechanism spontaneously breaks the electroweak symmetry, giving mass to:
    - W and Z bosons (weak force carriers)
    - Quarks and charged leptons (like electrons, muons, and tau)

- Gluons and photons do not interact with the Higgs field, so they remain massless.

Higgs Boson Properties

Property	Value
Symbol	H
Spin	0 (scalar particle)
Charge	0
Mass	~125 GeV/c^2 (measured at LHC)
Parity	Even
Decay Modes	$H \to b\bar{b}, H \to \gamma\gamma, H \to ZZ*, H \to WW*$